Lecture 3: Language Models

The backbone of NLP

Harvard

AC295/CS287r/CSCI E-115B

Chris Tanner

Don’t forget to start HW1 early!

Today’s lecture is brought to you by Teddy.

ANNOUNCEMENTS

- Keep an eye on the **HW1 Errata**, posted on Ed.
- I'll hold **Office Hours today** 2:30pm – 4:30pm
  - **Location**: out back of SEC 1st floor, or SEC 3.301-3.303 if weather isn't good

RECAP

a t e

61	74	65

- Default character-level representations aren't useful
- Simple document-level representations can be useful but have weaknesses
  - Context-insensitive ("the horse ate" = "ate the horse")
  - Curse of Dimensionality (vocab could be over 100k)
  - Orthogonality: no concept of semantic similarity at the word-level

$$\text{TFIDF} = f_{w_i} * log\left(\frac{\text{\# docs in corpus}}{\text{\# docs containing } w_i}\right)$$

Outline

- Language Modelling: what and why?
- Unigrams
- Bigrams
- Evaluation
- Beyond count-based models

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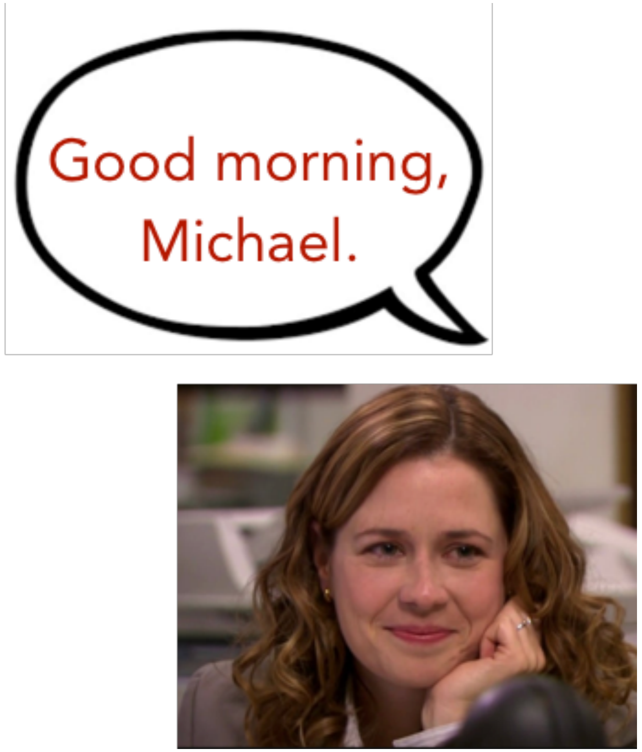
Language Modelling

A Language Model represents the language used by a given entity (e.g., a particular person, genre, or other well-defined class of text)



Language Modelling

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Spam

Not Spam

Language Modelling

A Language Model represents the language used by a given entity (e.g., a particular person, genre, or other well-defined class of text)

Language Modelling

FORMAL DEFINITION

A Language Model estimates the probability of any sequence of words

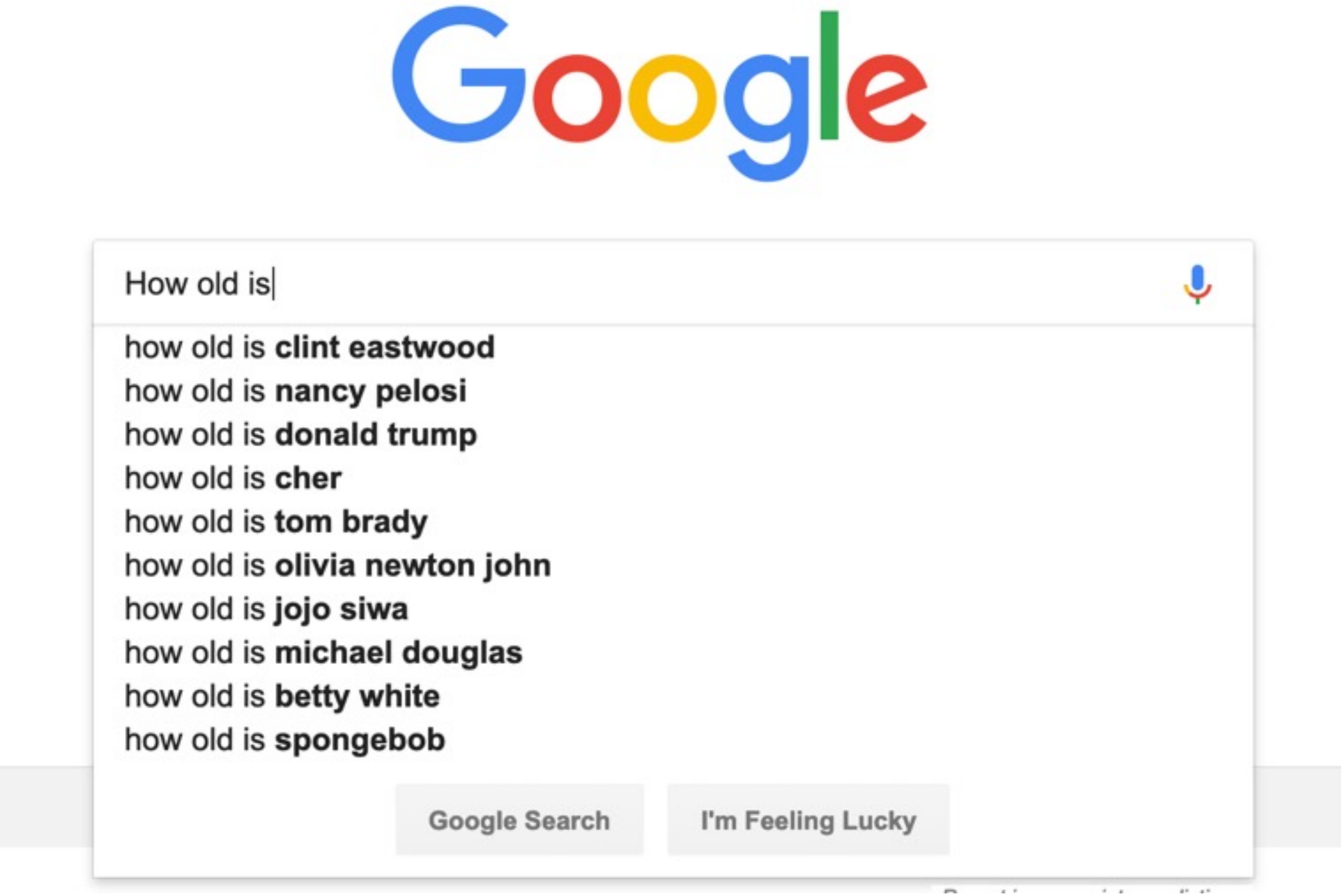
Let $\boldsymbol{X}$ = "Anqi was late for class"

w_1 w_2 w_3 w_4 w_5

$\mathrm{P}(\boldsymbol{X}) = P($"Anqi was late for class"$)$

Language Modelling

Generate Text

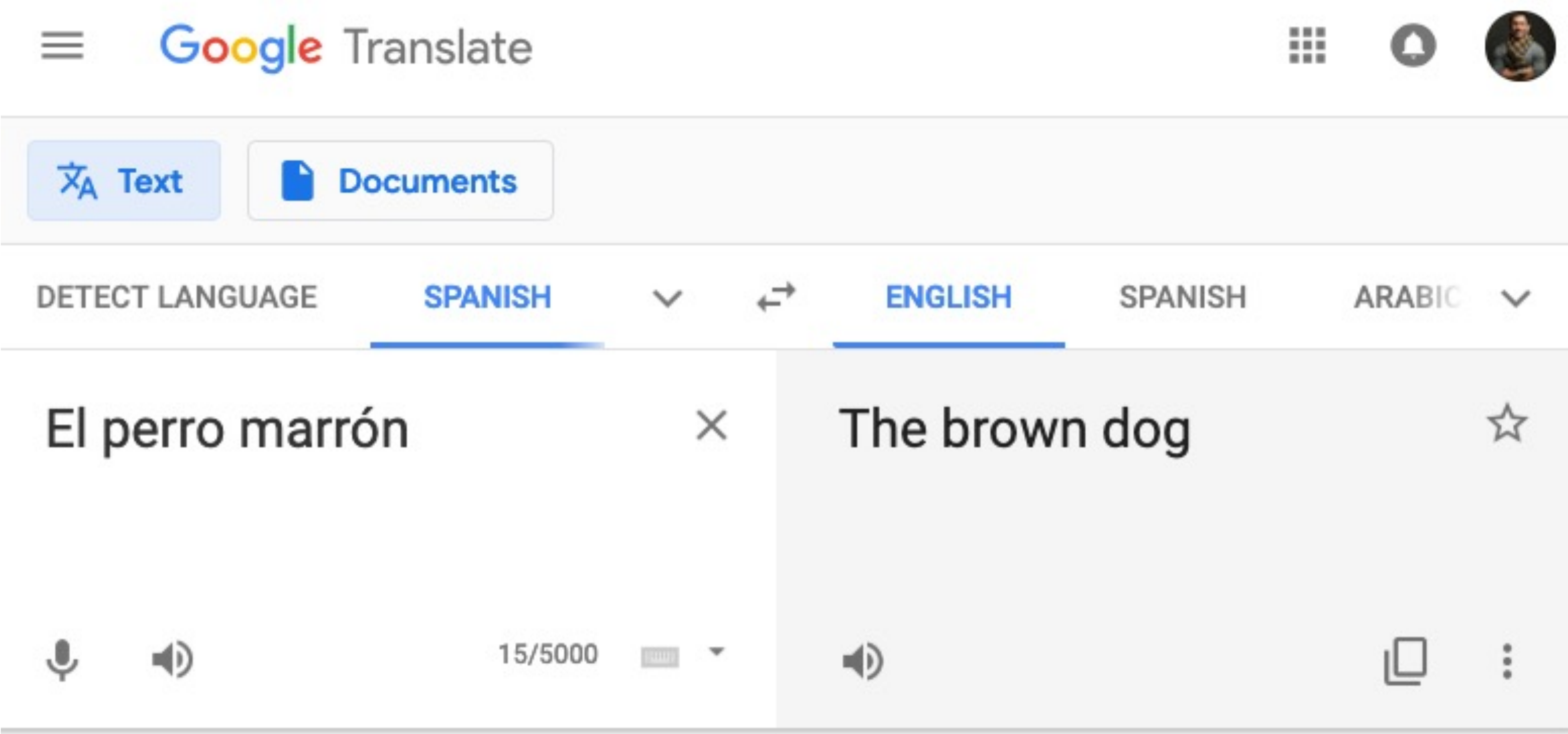


Language Modelling

Generate Text

Language Modelling

Generate Text



Language Modelling

"Drug kingpin El Chapo testified that he gave MILLIONS to Pelosi, Schiff & Killary. The Feds then closed the courtroom doors."

Fake News

Real News

Language Modelling

A Language Model is useful for:

Generating Text

- Auto-complete
- Speech-to-text
- Question-answering / chatbots
- Machine translation
- Summarization

Classifying Text

- Authorship attribution
- Detecting spam vs not spam
- Grammar Correction

And much more!

Language Modelling

Slide adapted from Luke Zettlemoyer @ UW 2018

Scenario: assume we have a finite vocabulary V

V^* represents the **infinite set** of strings/sentences that we could construct

e.g., V^*= {a, a dog, a frog, dog a, dog dog, frog dog, frog a dog, …}

Data: we have a training set of sentences $x \in V^*$

Problem: estimate a probability distribution:

$$\sum_{x \in V^*} p(x) = 1$$

$$p(the) = 10^{-2}$$

$$p(the, sun, okay) = 2.5x10^{-13}$$

$$p(waterfall, the, icecream) = 3.2x10^{-18}$$

Motivation

"Wreck a nice beach" vs "Recognize speech"

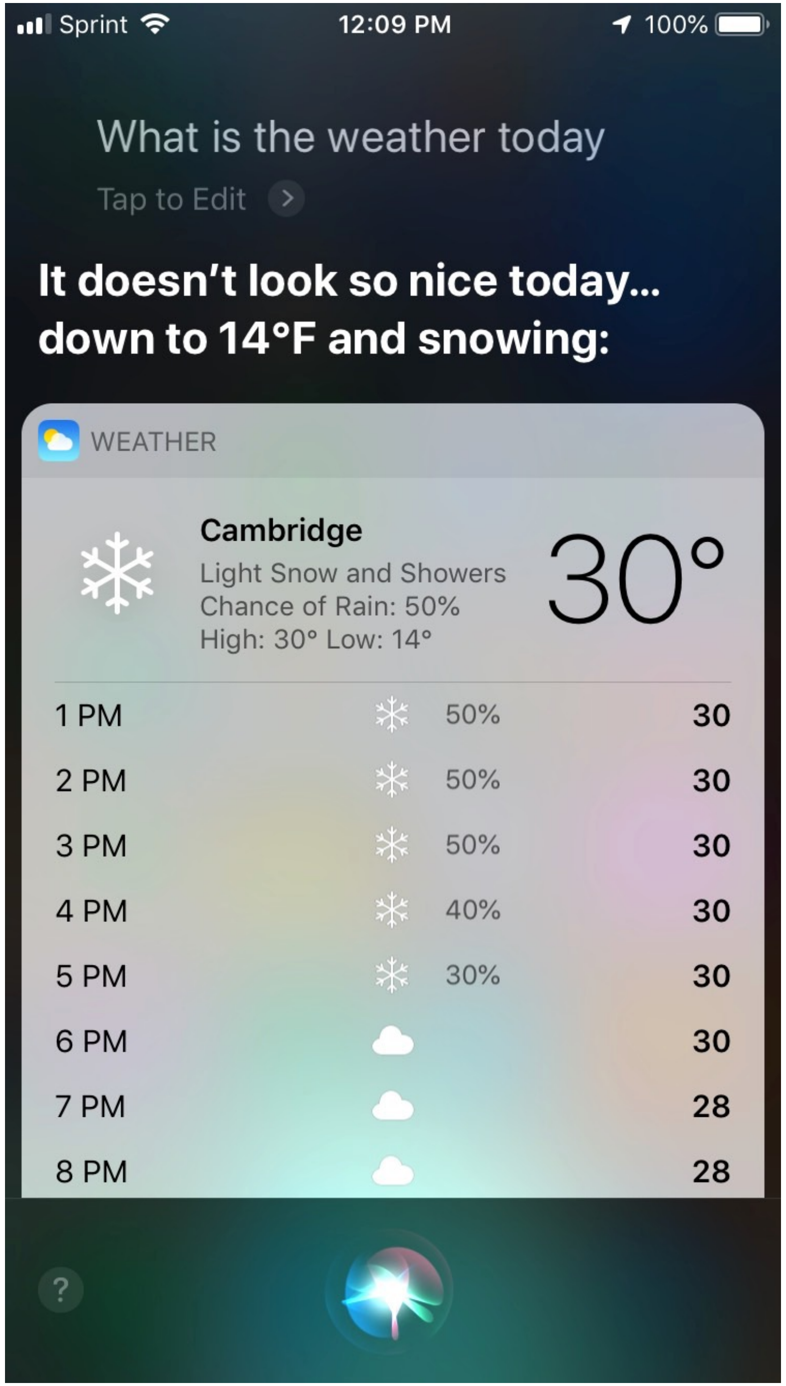
"I ate a cherry" vs "Eye eight uh Jerry!"

"What is the weather today?"

"What is the whether two day?"

"What is the whether too day?"

"What is the Wrether today?"



Language Modelling

How can we build a language model?

Outline

- Language Modelling: what and why?
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Important Terminology

a word <u>token</u> is a specific occurrence of a word in a text

a word <u>type</u> refers to the general form of the word, defined by its lexical representation

If our corpus were just "I ran and ran and ran", you'd say we have:

- 6 word **tokens** [I, ran , and , ran , and , ran]
- 3 word **types**: {I, ran, and}

Language Modelling

Naive Approach: unigram model

$$P(w_1, \dots, w_T) = \prod_{t=1}^{T} p(w_t)$$

Assumes each word is independent of all others.

Language Modelling

Naive Approach: unigram model

$$P(w_1, \ldots, w_T) = \prod_{t=1}^{T} p(w_t)$$

Assumes each word is independent of all others.

$$\mathrm{P}(\boldsymbol{w_1}, \boldsymbol{w_2}, \boldsymbol{w_3}, \boldsymbol{w_4}, \boldsymbol{w_5}) = \mathrm{P}(\boldsymbol{w_1}), \boldsymbol{P}(\boldsymbol{w_2}), \boldsymbol{P}(\boldsymbol{w_3})\boldsymbol{P}(\boldsymbol{w_4})\boldsymbol{P}(\boldsymbol{w_5})$$

Unigram Model

Let $\boldsymbol{X}$ = "Anqi was late for class"

$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$

Unigram Model

Let $\boldsymbol{X}$ = "Anqi was late for class"

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Let's say our corpus $\boldsymbol{d}$ has 100,000 words

word	# occurrences
Anqi	15
was	1,000
late	400
for	3,000
class	350

Unigram Model

Let $\boldsymbol{X}$ = "Anqi was late for class"

$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$

$$\mathrm{P}(\mathrm{w_i}) = \frac{n_{w_i}(\boldsymbol{d})}{n_{w_*}(\boldsymbol{d})}$$

Let's say our corpus $\boldsymbol{d}$ has 100,000 words

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$$n_{w_*}(\boldsymbol{d}) = 100{,}000$$

$n_{w_i}(\boldsymbol{d})$ = # of times word $\boldsymbol{w_i}$ appears in $\boldsymbol{d}$

$n_{w_*}(\boldsymbol{d})$ = # of times any word $\boldsymbol{w}$ appears in $\boldsymbol{d}$

Unigram Model

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$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$

$$\mathrm{P}(\mathrm{w_i}) = \frac{n_{w_i}(\boldsymbol{d})}{n_{w_*}(\boldsymbol{d})}$$

$$\mathrm{P}(\mathrm{Anqi}) = \frac{15}{100{,}000} = 0.00015$$

Let's say our corpus $\boldsymbol{d}$ has 100,000 words

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$$\mathrm{P}(\mathrm{w_i}) = \frac{n_{w_i}(\boldsymbol{d})}{n_{w_*}(\boldsymbol{d})}$$

$$\mathrm{P}(\mathrm{Anqi}) = \frac{15}{100{,}000} = 0.00015$$

$$\mathrm{P}(\mathrm{was}) = \frac{1{,}000}{100{,}000} = 0.01$$

Let's say our corpus $\boldsymbol{d}$ has 100,000 words

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$$n_{w_*}(\boldsymbol{d}) = 100{,}000$$

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$$\mathrm{P}(\mathrm{w_i}) = \frac{n_{w_i}(\boldsymbol{d})}{n_{w_*}(\boldsymbol{d})}$$

$$\mathrm{P}(\mathrm{Anqi}) = \frac{15}{100{,}000} = 0.00015$$

$$\mathrm{P}(\mathrm{was}) = \frac{1{,}000}{100{,}000} = 0.01$$

⋮

Let's say our corpus $\boldsymbol{d}$ has 100,000 words

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Unigram Model

Let $\boldsymbol{X}$ = "Anqi was late for class"

$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$

$$\mathrm{P}(\text{Anqi, was, late, for, class}) = \mathrm{P}(\text{Anqi})\mathrm{P}(\text{was})\ \mathrm{P}(\text{late})\ \mathrm{P}(\text{for})\ \mathrm{P}(\text{class})$$

Unigram Model

Let $\boldsymbol{X}$ = "Anqi was late for class"

$$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$$

$$\mathrm{P}(\mathrm{Anqi}, \mathrm{was}, \mathrm{late}, \mathrm{for}, \mathrm{class}) = \mathrm{P}(\mathrm{Anqi})\mathrm{P}(\mathrm{was})\ \mathrm{P}(\mathrm{late})\ \mathrm{P}(\mathrm{for})\ \mathrm{P}(\mathrm{class})$$

$$= 0.00015 * 0.01 * 0.004 * 0.03 * 0.0035$$

$$= 6.3 * 10^{-13}$$

Unigram Model

Let $\boldsymbol{X}$ = "Anqi was late for class"

$$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$$

$$\mathrm{P}(\text{Anqi, was, late, for, class}) = \mathrm{P}(\text{Anqi})\mathrm{P}(\text{was})\ \mathrm{P}(\text{late})\ \mathrm{P}(\text{for})\ \mathrm{P}(\text{class})$$

$$= 0.00015 * 0.01 * 0.004 * 0.03 * 0.0035$$

$$= 6.3 * 10^{-13}$$

This iterative approach is much more efficient than dividing by all possible sequences of length 5

Unigram Model

$$\mathrm{P}(\text{Anqi, was, late, for, class}) > \mathrm{P}(\text{Anqi, was, late, for, asdfjkl; })$$

$$\mathrm{P}(\text{Anqi, was, late, for, the}) >?\ \mathrm{P}(\text{Anqi, was, late, for, class})$$

$$\mathrm{P}(\text{Anqi, was, late, for, the}) <?\ \mathrm{P}(\text{Anqi, was, late, for, class})$$

UNIGRAM ISSUES?

?

UNIGRAM ISSUES?

1. Probabilities become too small

2. Out-of-vocabulary words <UNK>

3. Context doesn't play a role at all

$$P(\text{"Anqi was late for class"}) = P(\text{"class for was late Anqi"})$$

4. Sequence generation: What's the most likely next word?

Anqi was late for class _____

Anqi was late for class <u>the</u>

Anqi was late for class the <u>the</u>

UNIGRAM ISSUES?

Problem 1: Probabilities become too small

$$P(w_1, \ldots, w_T) = \prod_{t=1}^{T} p(w_t)$$

UNIGRAM ISSUES?

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$$P(w_1, \dots, w_T) = \prod_{t=1}^{T} p(w_t)$$

Solution:

$$\log \prod_{t=1}^{T} p(w_t) = \sum_{t=1}^{T} \log(p(w_i))$$

even $\log(10^{-100}) = -230.26$ is manageable

UNIGRAM ISSUES?

Problem 2: Out-of-vocabulary words <UNK>

$$p(\text{“}COVID19\text{”}) = 0$$

UNIGRAM ISSUES?

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$$p(\text{“}COVID19\text{”}) = 0$$

Solution: Smoothing

(give every word's count some inflation)

$$\mathrm{P}(\mathrm{w}) = \frac{n_w(\boldsymbol{d})}{n_{w_*}}$$

UNIGRAM ISSUES?

Problem 2: Out-of-vocabulary words <UNK>

$$p(\text{"}COVID19\text{"}) = 0$$

Solution: Smoothing

(give every word's count some inflation)

$$\mathrm{P(w)} = \frac{n_w(\boldsymbol{d})+\alpha}{n_{w_*}+\alpha|V|}$$

$$\mathrm{P(\text{"Anqi"})} = \frac{15+\alpha}{100{,}000 + \alpha|V|}$$

$|V|$ = the # of unique words types in vocabulary (including an extra 1 for <UNK>)

$$\mathrm{P(\text{"COVID19"})} = \frac{0+\alpha}{100{,}000 + \alpha|V|}$$

UNIGRAM ISSUES?

Two important notes:

1. Generally, α values are small (e.g., 0.5 – 2)
2. When a word w isn't found within the training corpus $\boldsymbol{d}$ you should replace it with <UNK> (or *U*)

$|V|$ = the # of unique words types in vocabulary (including an extra 1 for <UNK>)

$$P(\text{COVID19}) = \frac{0+\alpha}{100{,}000 + \alpha|V|}$$

UNIGRAM ISSUES?

Problems 3 and 4: Context doesn't play a role at all

$$P(\text{"Anqi was late for class"}) = P(\text{"class for was late Anqi"})$$

Question: How can we factor in context?

UNIGRAM ISSUES?

Easiest Approach:

Instead of words being completely independent, condition each word on its immediate predecessor

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Bigram LM

Look at *pairs* of consecutive words

Let $\boldsymbol{X}$ = "Anqi was late for class"

$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$

Bigram LM

Look at *pairs* of consecutive words

probability

Let $\boldsymbol{X}$ = "Anqi was late for class"

w_1 w_2 w_3 w_4 w_5

$\mathrm{P}(\boldsymbol{X}) = P(\text{was}|\text{Anqi})$

Bigram LM

Look at *pairs* of consecutive words

probability

Let $\boldsymbol{X}$ = "Anqi $\boxed{\text{was late}}$ for class"

w_1 w_2 w_3 w_4 w_5

$$\mathrm{P}(\boldsymbol{X}) = P(\text{was}|\text{Anqi})P(\text{late}|\text{was})$$

Bigram LM

Look at *pairs* of consecutive words

probability

Let $\boldsymbol{X}$ = "Anqi was $\boxed{\text{late for}}$ class"

w_1 w_2 w_3 w_4 w_5

$$\mathrm{P}(\boldsymbol{X}) = P(\text{was}|\text{Anqi})P(\text{late}|\text{was})P(\text{for}|\text{late})$$

Bigram LM

Look at *pairs* of consecutive words

probability

Let $\boldsymbol{X}$ = "Anqi was late $\boxed{\text{for class}}$"

w_1 w_2 w_3 w_4 w_5

$$\mathrm{P}(\boldsymbol{X}) = P(\text{was}|\text{Anqi})P(\text{late}|\text{was})P(\text{for}|\text{late})P(\text{class}|\text{for})$$

Bigram LM

You calculate each of these probabilities by simply counting the occurrences

probability

Let $\boldsymbol{X}$ = "Anqi was late for class"

w_1 w_2 w_3 w_4 w_5

$$\mathrm{P}(\boldsymbol{X}) = P(\text{was}|\text{Anqi})P(\text{late}|\text{was})P(\text{for}|\text{late})P(\text{class}|\text{for})$$

Bigram Model

Let $\boldsymbol{X}$ = "Anqi was late for class"

$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$

$$\mathrm{P}(\mathrm{w}'|w) = \mathrm{P}(\text{"w,w}'\text{"}) = \frac{n_{w,w'}(\boldsymbol{d})}{n_{w,w*}(\boldsymbol{d})}$$

$n_{w,w'}(\boldsymbol{d})$ = # of times words $\boldsymbol{w}$ and $\boldsymbol{w}'$ appear together as a bigram in $\boldsymbol{d}$

$n_{w,w*}(\boldsymbol{d})$ = # of times word $\boldsymbol{w}$ is the first token of a bigram in $\boldsymbol{d}$

Bigram Model

Let $\boldsymbol{X}$ = "Anqi was late for class"
$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5$

$$P(w'|w) = P(\text{"w,w'"}) = \frac{n_{w,w'}(\boldsymbol{d})}{n_{w,w*}(\boldsymbol{d})}$$

$$P(\text{class}|\text{for}) = P(\text{for}, \text{class}) = \frac{12}{3{,}000}$$

Let's say our corpus $\boldsymbol{d}$ has 100,000 words

word	# occurrences
Anqi	15
was	1,000
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$$n_{w_*}(\boldsymbol{d}) = 100{,}000$$

$n_{w,w'}(\boldsymbol{d})$ = # of times words $\boldsymbol{w}$ and $\boldsymbol{w}'$ appear together as a bigram in $\boldsymbol{d}$

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BIGRAM ISSUES?

?

BIGRAM ISSUES?

1. Out-of-vocabulary bigrams are 0 → kills the overall probability

2. Could always benefit from more context but sparsity is an issue (e.g., rarely seen 5-grams)

3. Storage becomes a problem as we increase the window size

4. No semantic information conveyed by counts (e.g., vehicle vs car)

BIGRAM ISSUES?

Problem 1: Out-of-vocabulary bigrams

Our current bigram probabilities:

$$P(w,w') = \frac{n_{w,w'}(\boldsymbol{d})}{n_{w,w*}(\boldsymbol{d})}$$

Q: What should we do?

How we smoothed unigrams:

$$P(w) = \frac{n_w(\boldsymbol{d})+\alpha}{n_{w_*}+\alpha|V|}$$

$|V|$ = the # of unique words types in vocabulary (including an extra 1 for <UNK>)

BIGRAM ISSUES?

Problem 1: Out-of-vocabulary bigrams

Imagine our current string x includes "COVID19 harms ribofliptonik …"

In our training corpus d, we've never seen:

"COVID19 harms" or "harms ribofliptonik"

But we've seen the unigram "harms", which provides useful information:

BIGRAM ISSUES?

Problem 1: Out-of-vocabulary bigrams

Solution: unigram-backoff for smoothing

$$P(\text{"w,w}'\text{"}) = \frac{n_{w,w'}(\boldsymbol{d}) + \beta * P(\mathrm{w}')}{n_{w,w*}(\boldsymbol{d}) + \beta}$$

$$P(\mathrm{w}') = \frac{n_{w'}(\boldsymbol{d}) + \alpha}{n_{w_*} + \alpha|V|}$$

$|V|$ = the # of unique words types in vocabulary (including an extra 1 for <UNK>)

BIGRAM ISSUES?

Problem 1: Out of vocabulary bigrams

Solution: unigram-backoff for smoothing

Our model is properly parameterized with $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

So, instead of calculating the probability of text, we are actually interested in fixing the parameters at particular values and determining the likelihood of the data.

$|V|$ = the # of unique words types in vocabulary
(including an extra 1 for <UNK>)

BIGRAM ISSUES?

For a fixed $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$:

$$\theta(\text{"w,w'"}) = \frac{n_{w,w'}(\boldsymbol{d}) + \beta * \theta(\text{w}')}{n_{w,w*}(\boldsymbol{d}) + \beta}$$

$$\theta(\text{w}') = \frac{n_{w'}(\boldsymbol{d}) + \alpha}{n_{w_*} + \alpha|V|}$$

$|V|$ = the # of unique words types in vocabulary
(including an extra 1 for <UNK>)

IMPORTANT:

It is common to pad sentences with <S> tokens on each side, which serve as boundary markers. This helps LMs learn the transitions between sentences.

Let $\boldsymbol{X}$ = "I ate. Did you?"

$w_1 \; w_2 \; w_3 \; w_4$

→

$\boldsymbol{X}$ = "<S> I ate <S> Did you? <S>"

$w_1 \; w_2 \; w_3 \; w_4 \; w_5 \; w_6 \; w_7$

Generation

- We can also use these LMs to **generate** text
- Generate the very first token manually by making it be <S>
- Then, generate the next token by sampling from the probability distribution of possible next tokens (the set of possible *next* tokens sums to 1)
- When you generate be <S> again, that represents the end of the current sentence

Example of Bigram generation

- Force a <S> as the first token
- Of the bigrams that start with <S>, probabilistically pick one based on their likelihoods
- Let's say the chosen bigram was <S>_The
- Repeat the process, but now condition on "The". So, perhaps the next select Bigram is "The_dog"
- The sentence is complete when you generate a bigram whose second half is <S>

Imagine more context

Language Modelling

Better Approach: n-gram model

$$P(x_1, \dots, x_T) = \prod_{t=1}^{T} p(x_t | x_{t-1}, \dots, x_1)$$

Let's factor in context (in practice, a window of size **n**)

Language Modelling

Better Approach: n-gram model

$$P(x_1, \dots, x_T) = \prod_{t=1}^{T} \underbrace{p(x_t | x_{t-1}, \dots, x_1)}_{\text{The likelihood of any event occurring hinges upon all prior events occurring}}$$

Language Modelling

Better Approach: n-gram model

$$P(x_1, \ldots, x_T) = \overbrace{\prod_{t=1}^{T} \underbrace{p(x_t | x_{t-1}, \ldots, x_1)}_{\text{The likelihood of any event occurring hinges upon all prior events occurring}}}^{\text{This compounds for all subsequent events, too}}$$

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Evaluation

N-gram models seem useful, but how can we measure how good they are?

Can we just use the likelihood values?

Evaluation

Almost!

The likelihood values aren't adjusted for the length of sequences, so we would need to normalize by the sequence lengths.

$$H(C_{test}) = \frac{1}{N}\sum_{i=1}^{n} \log_2(p(w_i))$$

Perplexity

The best language model is one that
best predicts an unseen test set

Perplexity, denoted as PP, is the inverse probability of the test set, normalized by the number of words.

$$PP(w_1, \dots, w_N) = p(w_1, w_2, \dots, w_N)^{-1/N}$$

$$= \sqrt[N]{\frac{1}{p(w_1, w_2, \dots, w_N)}}$$

Perplexity

Perplexity is also equivalent to the exponentiated, per-word cross-entropy

$$PP(w_1, \ldots, w_N) = p(w_1, w_2, \ldots, w_N)^{-1/N}$$

$$= \sqrt[N]{\frac{1}{p(w_1, w_2, \ldots, w_N)}}$$

$$= 2^{-l}, \text{ where } \mathrm{l} = \frac{1}{N} \sum_{i=1}^{n} \log_2(p(w_i))$$

Perplexity

Very related to entropy, **perplexity** measures the **uncertainty** of the model for a particular dataset. So, very high perplexity scores correspond to having tons of uncertainty (which is bad).

Entropy represents the **average** number of bits needed to represent each word.

Perplexity represents the branching factor needed to predict each next word. That is, the more branches (aka bits) at each step, the more uncertainty there is, meaning the worse the model.

Perplexity

Good models tend to have perplexity scores around 40-100 on large, popular corpora.

If our model assumed a uniform distribution of words, then our perplexity score would be:

$|V|$ = the # of unique word types

Perplexity

Example: let our corpus X have only 3 unique words: {the, dog, ran} but our particular text has a length of N.

$$PP(w_1, \dots, w_N) = p(w_1, w_2, \dots, w_N)^{-1/N}$$

$$= \sqrt[N]{\frac{1}{p(w_1, w_2, \dots, w_N)}}$$

$$= \sqrt[N]{\frac{1}{\left(\frac{1}{3}\right)^N}} = \sqrt[N]{3^N} = 3$$

Perplexity

More generally, if we have M unique words for a sequence of length N.

$$PP(X) = \sqrt[N]{\frac{1}{\left(\frac{1}{M}\right)^N}} = \sqrt[N]{M^N} = M$$

Perplexity

Example perplexity scores: when trained on a corpus of 38 million words and tested on 1.5 million words:

model	perplexity
unigram	962
bigram	170
trigram	109

Evaluation

Very Important:

- Any given LM must be able to generate the **test set (**at least**)**. Otherwise, it cannot be fairly evaluated (OOV problem).
- When comparing multiple LMs to each other, their vocabularies must be the same (e.g., words, sub-words, characters).

Slide adapted from or inspired by Graham Neubig's CMU NLP 2021

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Remaining Issues

1. More context while avoiding sparsity, storage, and compute issues

2. No semantic information conveyed by counts (e.g., vehicle vs car)

New goals!

3. Cannot leverage non-consecutive patterns

Dr. West ____

Occurred 25 times

Dr. Cornell West ____

Occurred 3 times

4. Cannot capture combinatorial signals (i.e., non-linear prediction)

P(Chef cooked food)

P(Chef ate food)

P(Customer cooked food)

P(Customer ate food)

Slide adapted from or inspired by Graham Neubig's CMU NLP 2021

Featurized Model

Instead of counts, let's move toward having **words** represented as <u>features</u>

features ≪ # of words in vocab

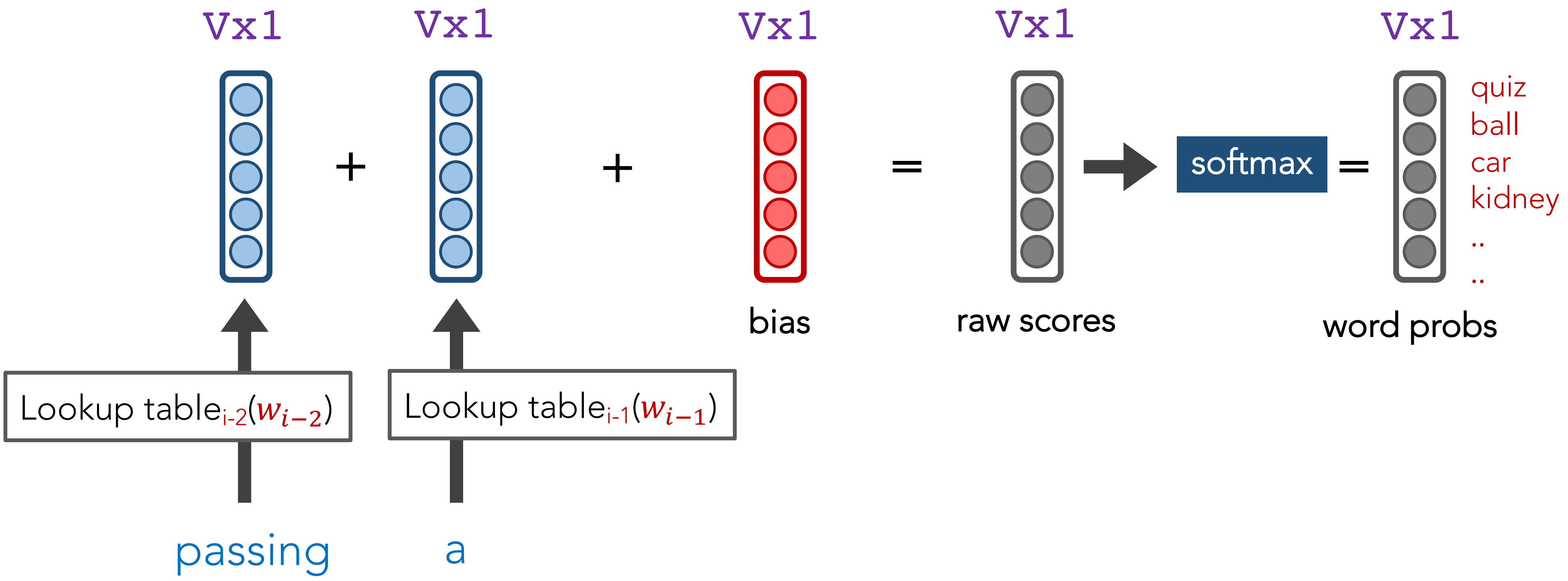
We can develop a very simple linear model that calculates word probabilities

Slide adapted from or inspired by Graham Neubig's CMU NLP 2021

Featurized Model

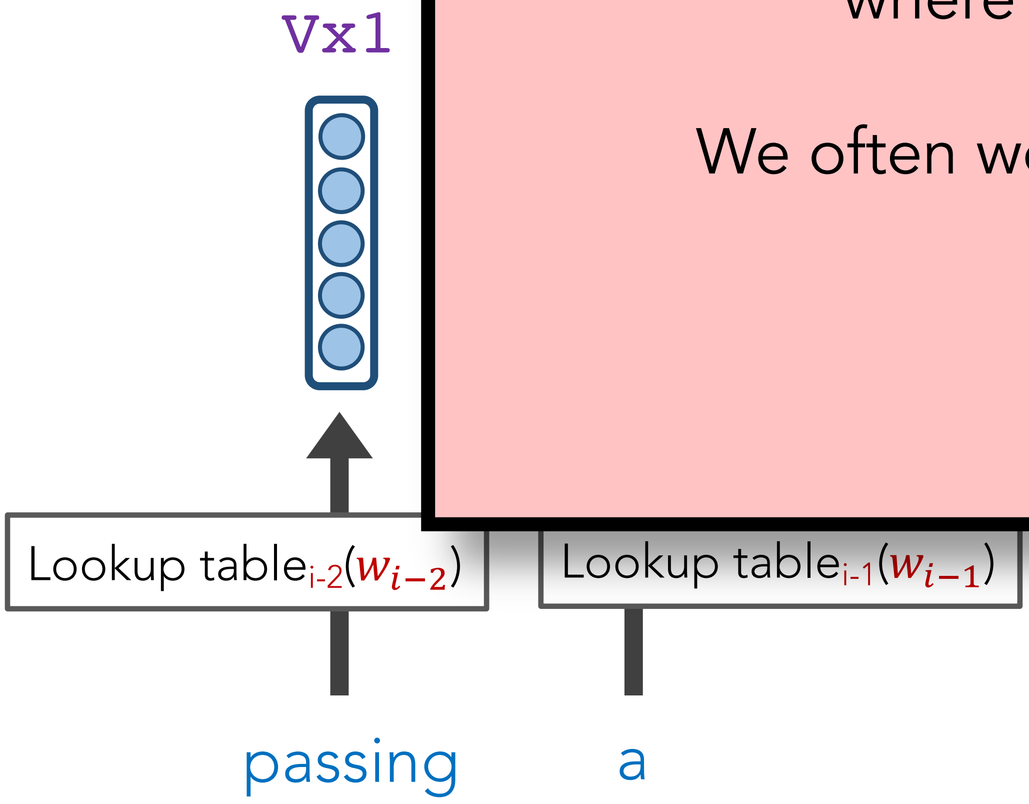
"passing a ______"

w_{i-2} w_{i-1} w_i



Slide adapted from or inspired by Graham Neubig's CMU NLP 2021

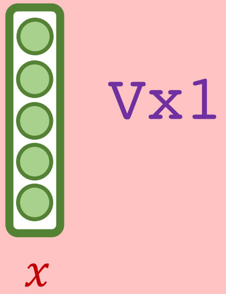
Featurized Model



A "lookup table" is trivial.

It simply converts each unique word to an index $i \in V$, where V is the size of our vocabulary.

We often work with the one-hot version of it, x:



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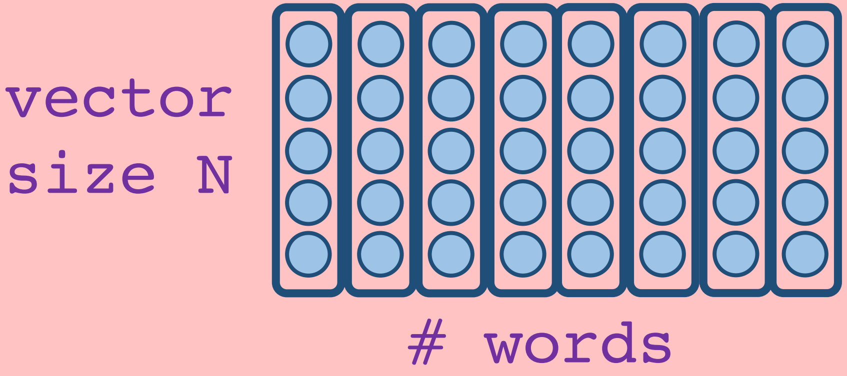
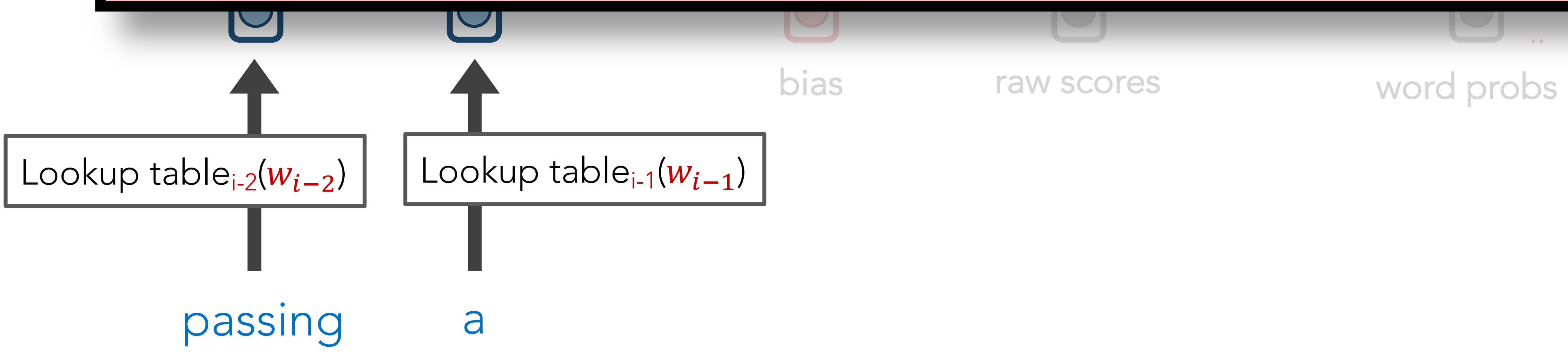
Featurized Model

Embedding/ feature matrix $\boldsymbol{v}$ is an "input word matrix". The i^{th} column of $\boldsymbol{v}$ corresponds to each unique word w_i



Can retrieve Embedding v via:
- Slicing the index, or
- Matrix multiply

$$v_i = \boldsymbol{v} x_i$$



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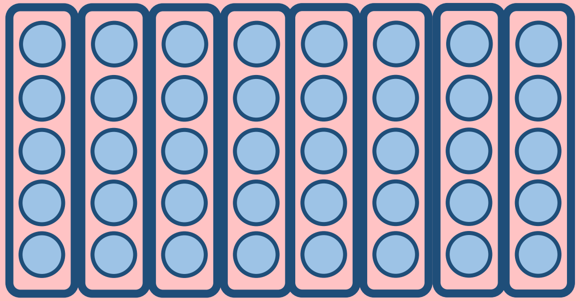
Featurized Model

Embedding/ feature matrix $\boldsymbol{v}$ is an "input word matrix". The i^{th} column of $\boldsymbol{v}$ corresponds to each unique word w_i

vector size N



words

Can retrieve Embedding v via:

- Slicing the index, or
- Matrix multiply

$$v_i = \boldsymbol{v} x_i$$

Nx1 = NxV * Vx1

Lookup table$_{i-2}$(w_{i-2})

Lookup table$_{i-1}$(w_{i-1})

passing

a

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Featurized Model

Train the model using gradient descent:

- Use our output probabilities
- Calculate the cross-entropy loss
- Use backprop to calculate gradients
- Update the 2 look-up table weights and bias via GD

Unknown Words

- We still need to handle UNK words. Always.
- Language is always evolving
- Zipfian distribution
- Larger vocabularies require more memory and compute time

How can we handle UNK words in a neural model?

Unknown Words

- Common ways:
  - Frequency threshold (e.g., UNK <= 2)
  - Remove bottom N%

Remaining Issues

1. More context while avoiding sparsity, storage, and compute issues

2. No semantic information conveyed by counts (e.g., vehicle vs car)

New goals!

3. Cannot leverage non-consecutive patterns

Dr. West ____

Occurred 25 times

Dr. Cornell West ____

Occurred 3 times

4. Cannot capture combinatorial signals (i.e., non-linear prediction)

P(Chef cooked food)

P(Chef ate food)

P(Customer cooked food)

P(Customer ate food)

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UP NEXT

We clearly need:

- denser representations, not |V|
- semantic information
- non-linear power

Neural models, here we come!