Comparison of Ridge and Lasso

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Ridge, LASSO - Computational complexity

Solution to ridge regression:

$$\beta = (X^T X + \lambda I)^{-1} X^T Y$$

The solution to the LASSO regression:

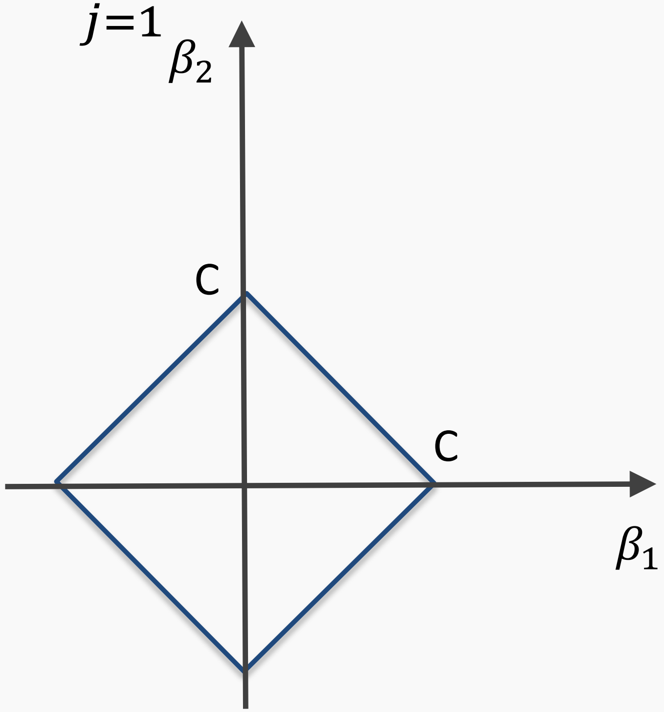
LASSO has no conventional analytical solution, as the L1 norm has no derivative at 0. We can, however, use the concept of subdifferential or subgradient to find a manageable expression.

The Geometry of Regularization (LASSO)

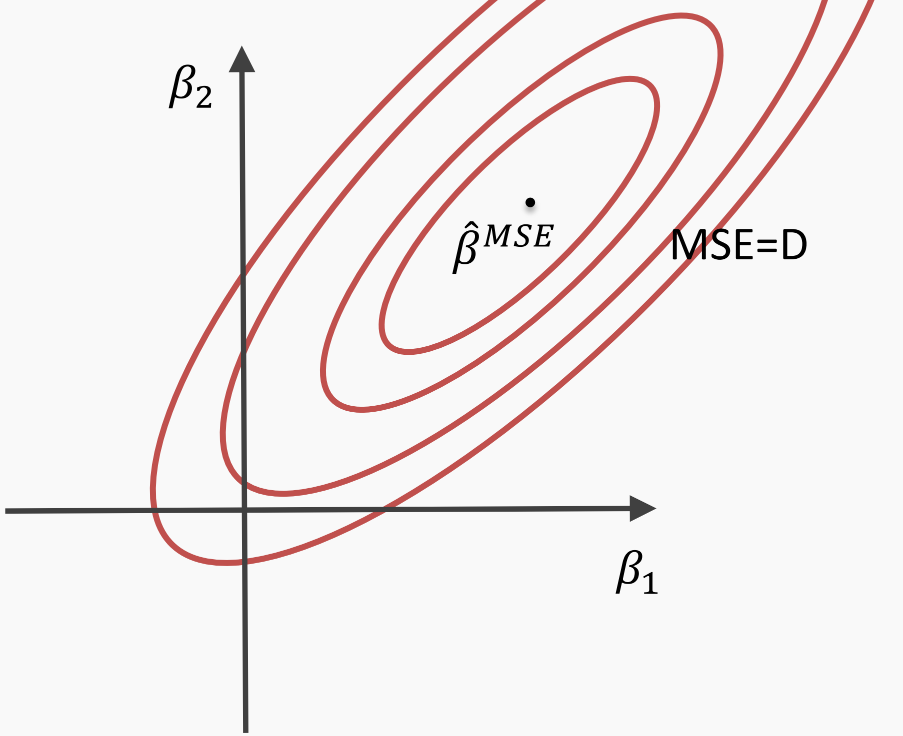
$$L_{LASSO}(\boldsymbol{\beta}) = \frac{1}{n}\sum_{i=1}^{n} \left|y_i - \boldsymbol{\beta}^T\boldsymbol{x}\right|^2 + \lambda\sum_{j=1}^{J}|\beta_j|$$

$$\widehat{\boldsymbol{\beta}}^{LASSO} = \operatorname{argmin} L_{LASSO}(\boldsymbol{\beta})$$

$$\lambda\sum_{j=1}^{J}\left|\,\hat{\beta}_j^{LASSO}\right| = C$$



$$\frac{1}{n}\sum_{i=1}^{n}\left|y_i - \widehat{\boldsymbol{\beta}}^{LASSO^T}\boldsymbol{x}\right|^2 = D$$



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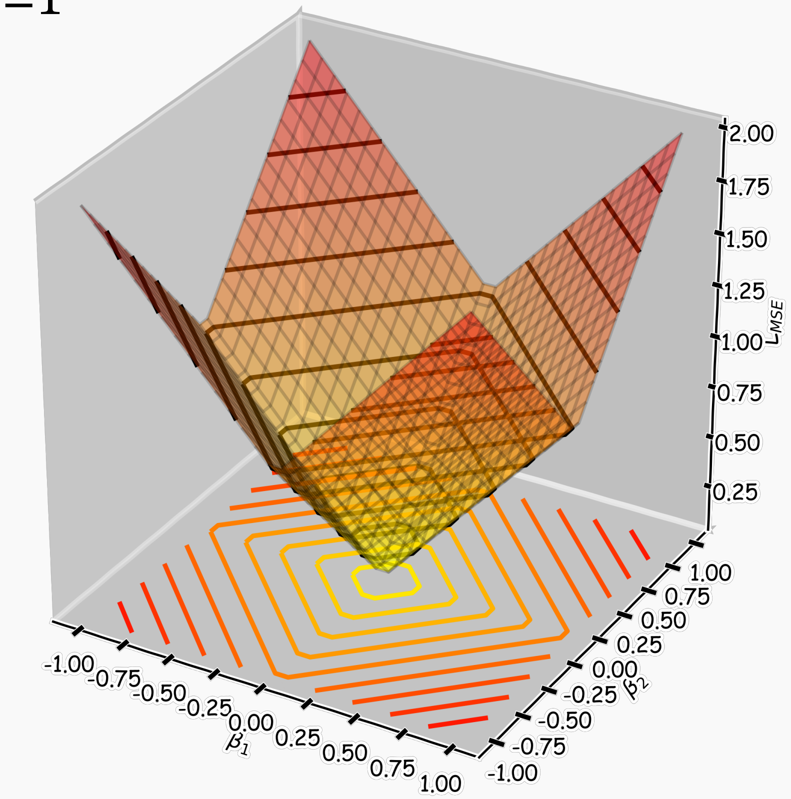
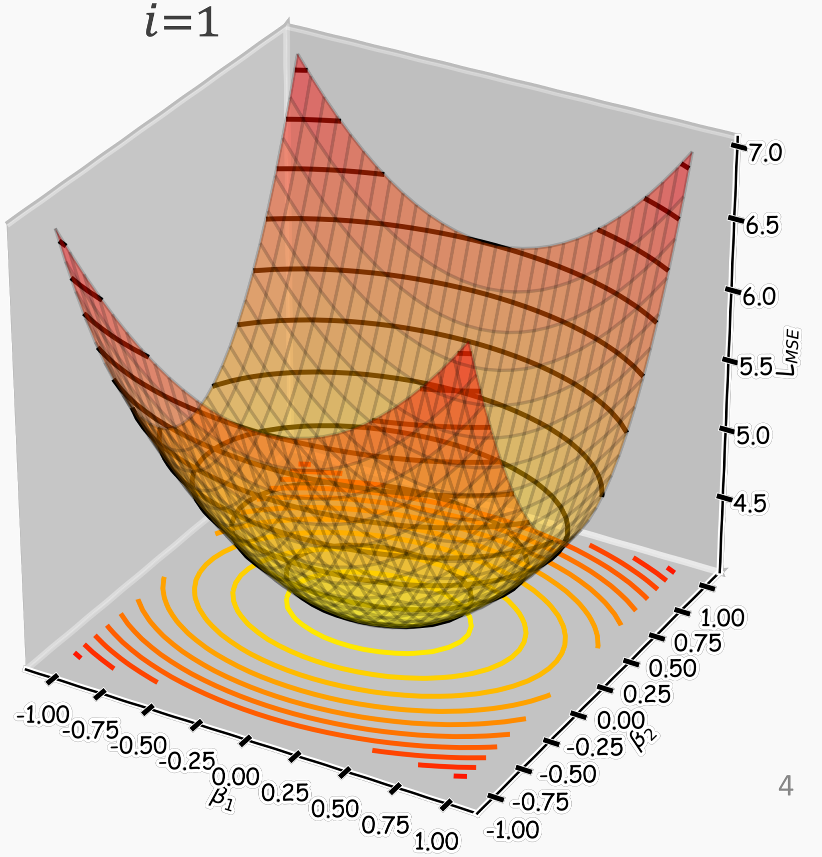
$$\widehat{\boldsymbol{\beta}}^{LASSO} = \operatorname{argmin} L_{LASSO}(\boldsymbol{\beta})$$

$$L_1 = \lambda\sum_{j=1}^{J}\left|\hat{\beta}_j^{LASSO}\right|$$



$$L_{MSE}(\boldsymbol{\beta}) = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \boldsymbol{\beta}^T\boldsymbol{x}\right|^2$$

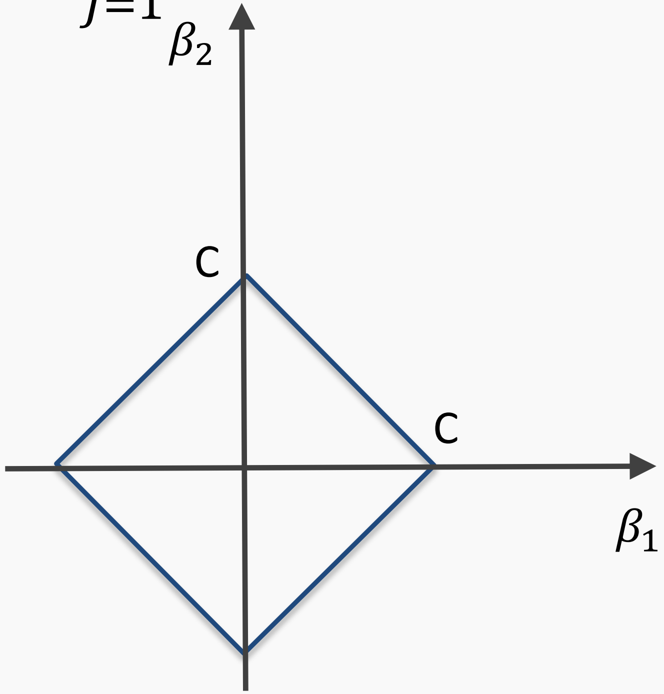


The Geometry of Regularization (LASSO)

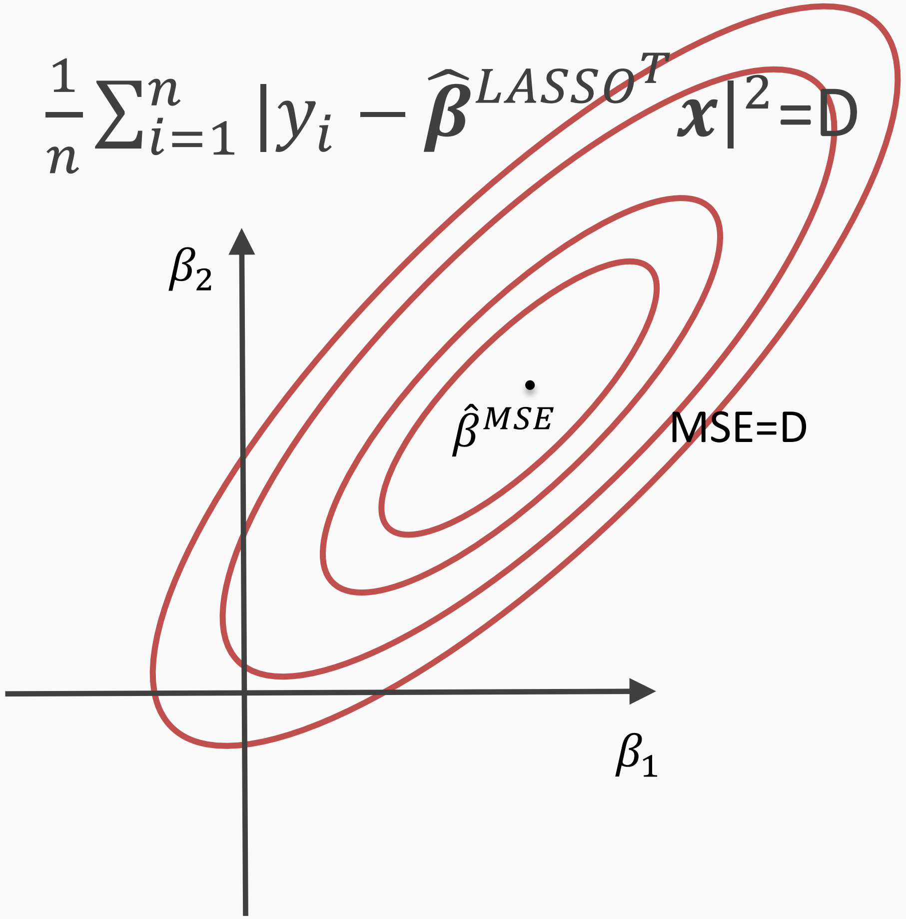
$$L_{LASSO}(\boldsymbol{\beta}) = \frac{1}{n}\sum_{i=1}^{n} \left|y_i - \boldsymbol{\beta}^T \boldsymbol{x}\right|^2 + \lambda \sum_{j=1}^{J} |\beta_j|$$

$$\widehat{\boldsymbol{\beta}}^{LASSO} = \text{argmin}\, L_{LASSO}(\boldsymbol{\beta})$$

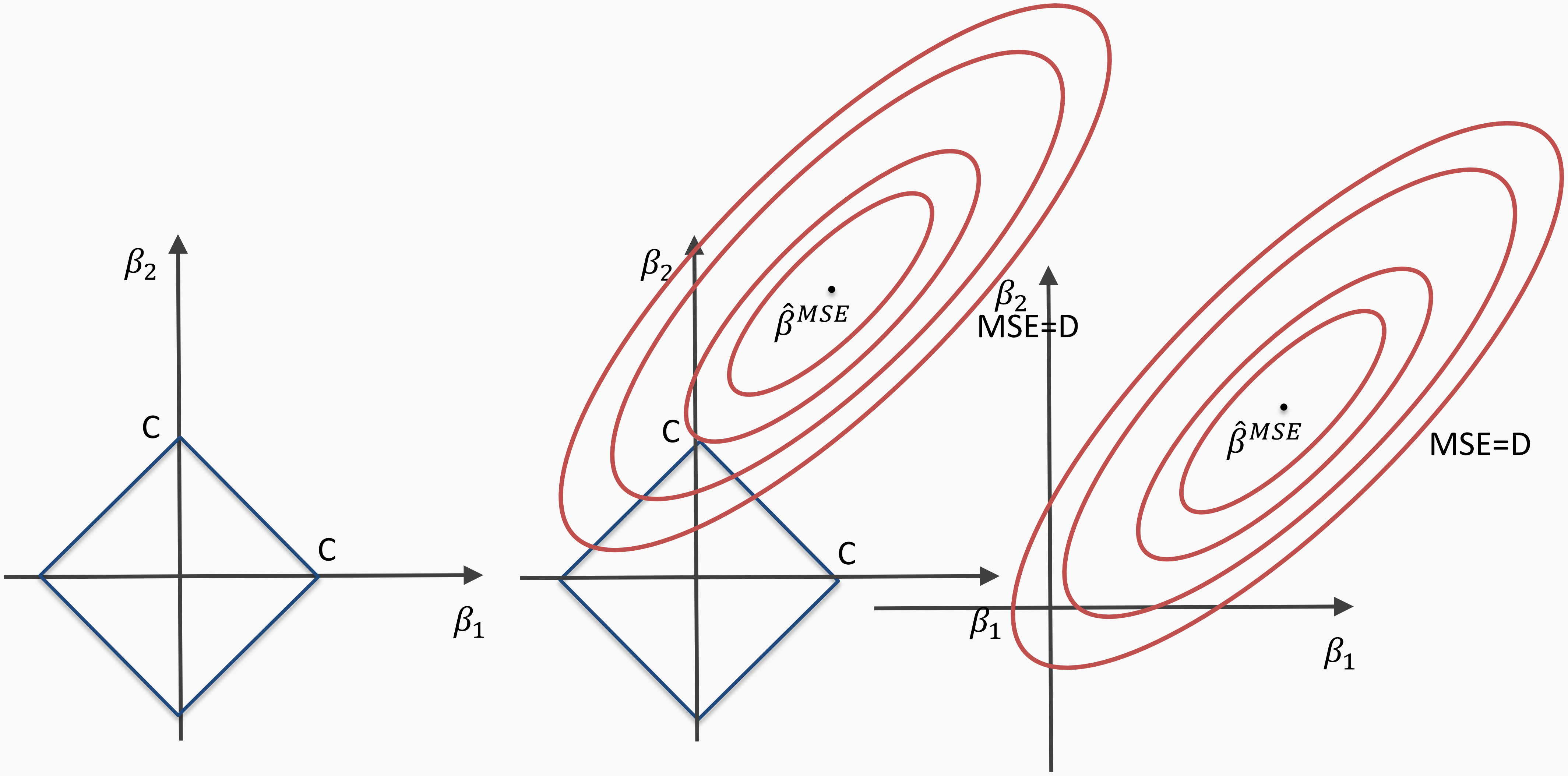
$$\lambda \sum_{j=1}^{J} \left| \hat{\beta}_j^{LASSO} \right| = C$$



$$\frac{1}{n}\sum_{i=1}^{n} \left|y_i - \widehat{\boldsymbol{\beta}}^{LASSO^T} \boldsymbol{x}\right|^2 = D$$



The Geometry of Regularization (LASSO)

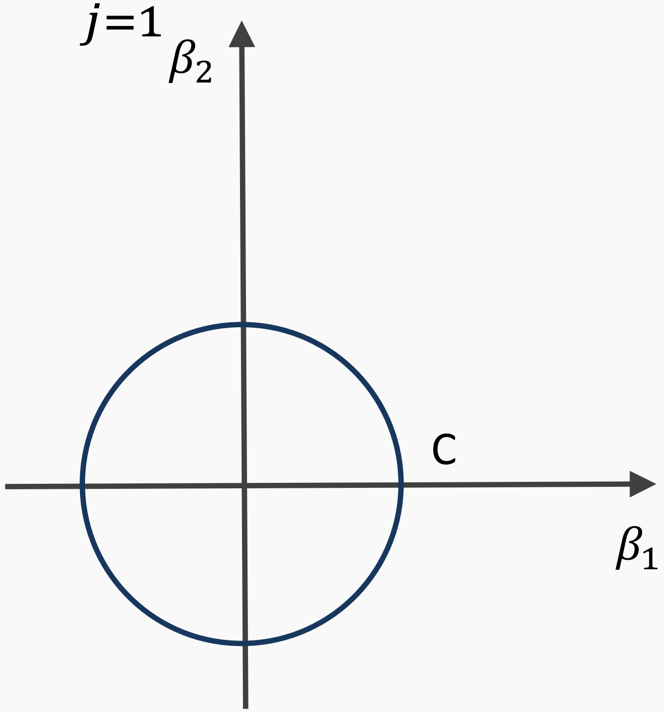


The Geometry of Regularization (Ridge)

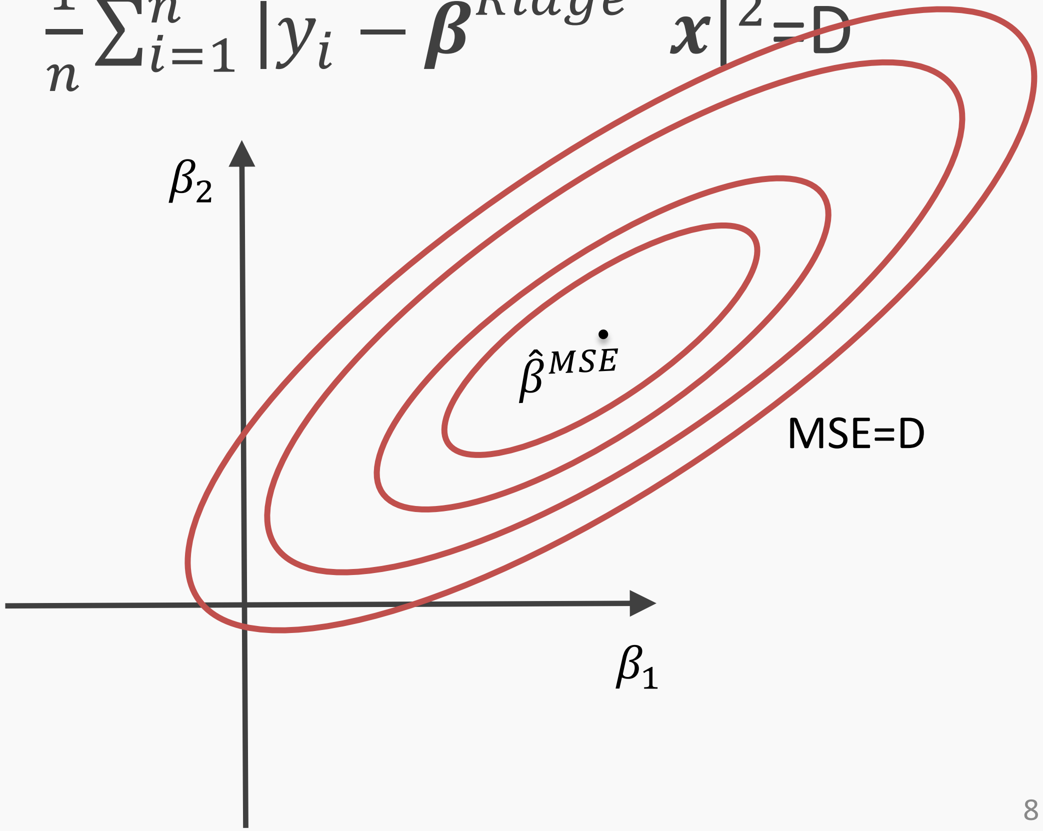
$$L_{Ridge}(\boldsymbol{\beta}) = \frac{1}{n}\sum_{i=1}^{n}\left|y_i - \boldsymbol{\beta}^T\boldsymbol{x}\right|^2 + \lambda\sum_{j=1}^{J}(\beta_j)^2$$

$$\widehat{\boldsymbol{\beta}}^{Ridge} = \operatorname{argmin} L_{Ridge}(\boldsymbol{\beta})$$

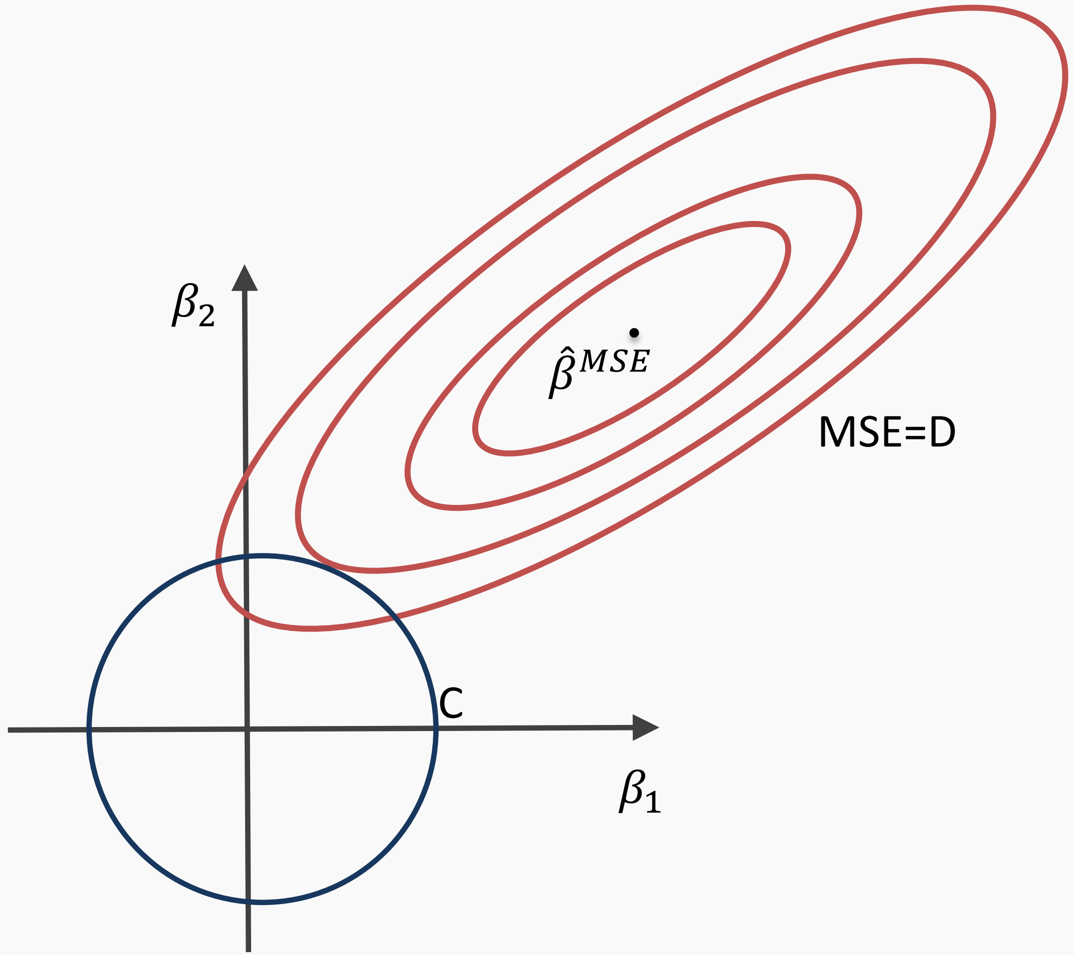
$$\lambda\sum_{j=1}^{J}\left|\hat{\beta}_j^{Ridge}\right|^2 = C$$



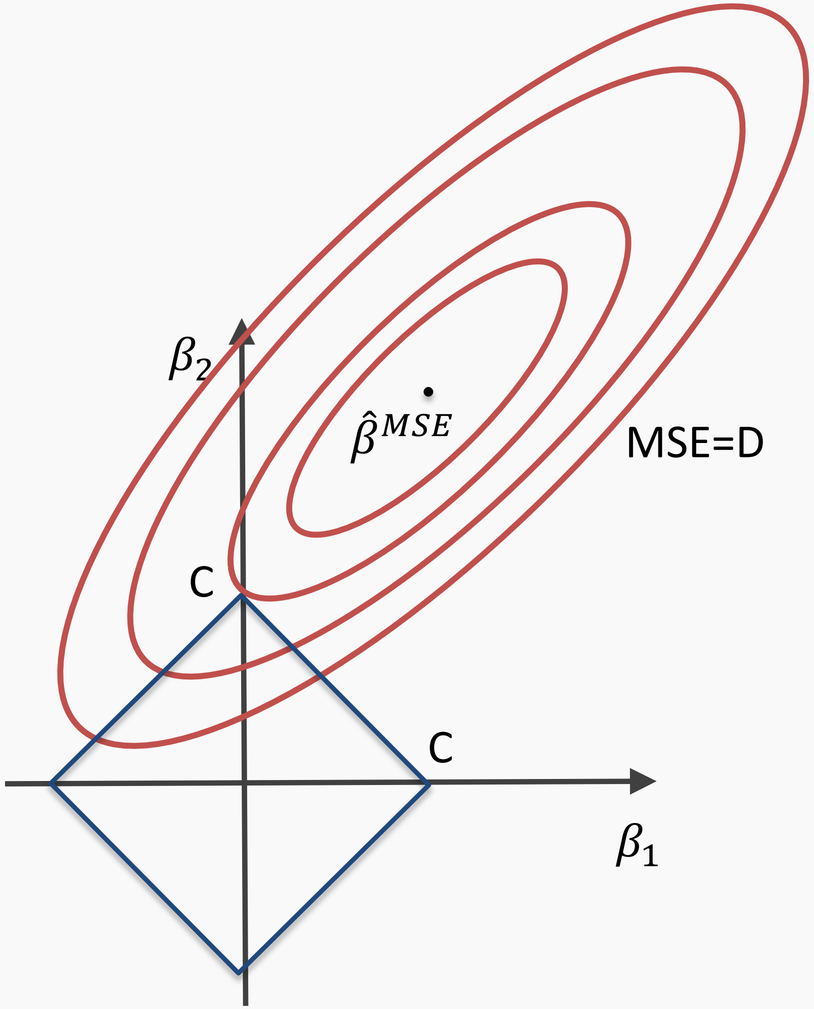
$$\frac{1}{n}\sum_{i=1}^{n}\left|y_i - \widehat{\boldsymbol{\beta}}^{Ridge^T}\boldsymbol{x}\right|^2 = \text{D}$$

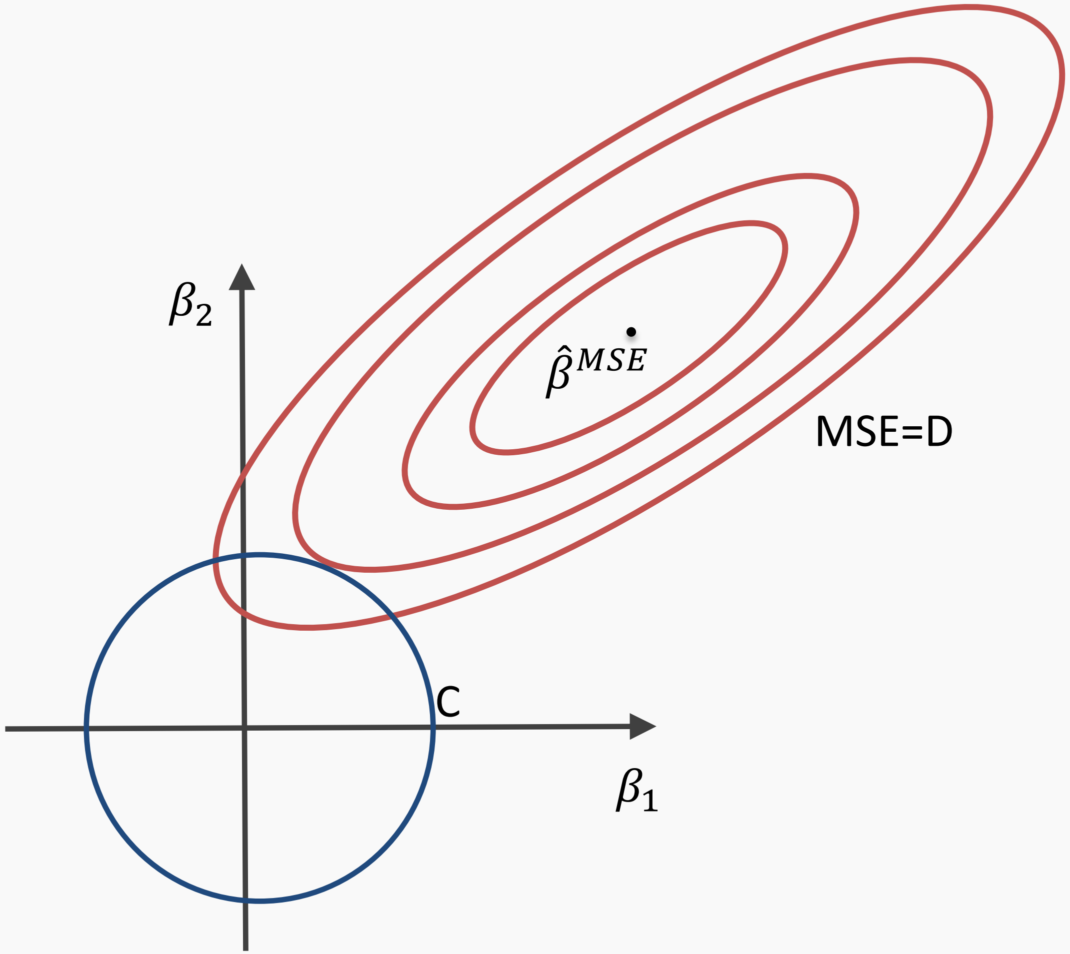


The Geometry of Regularization (Ridge)

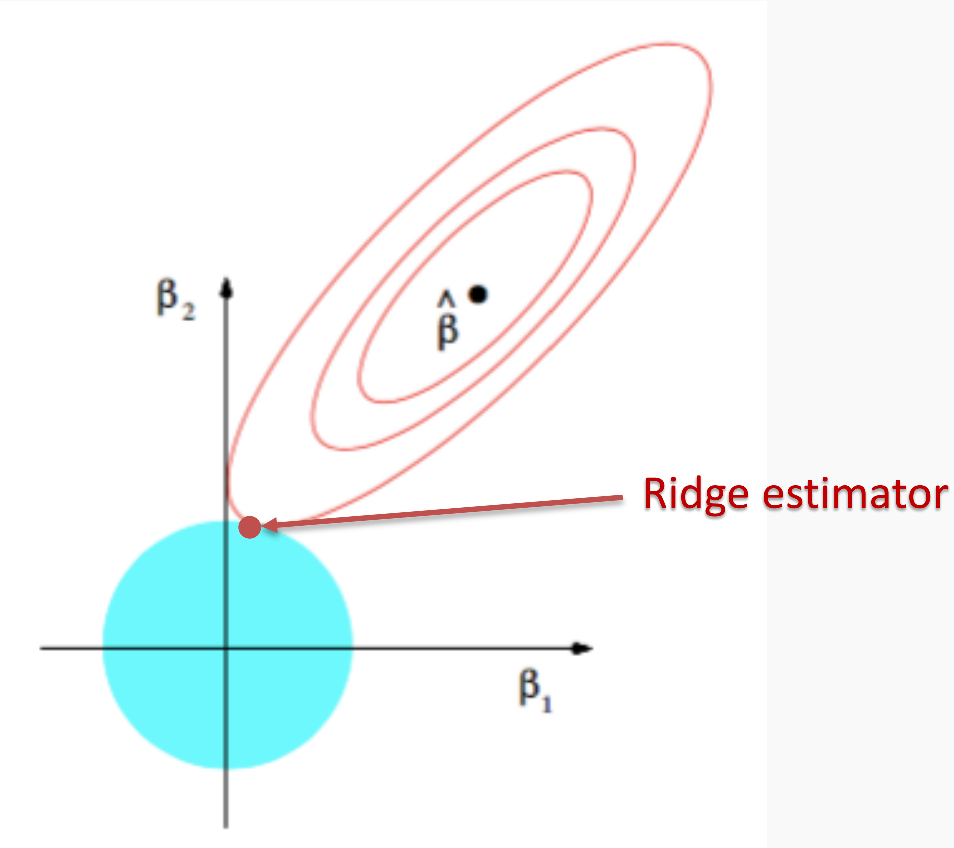


The Geometry of Regularization

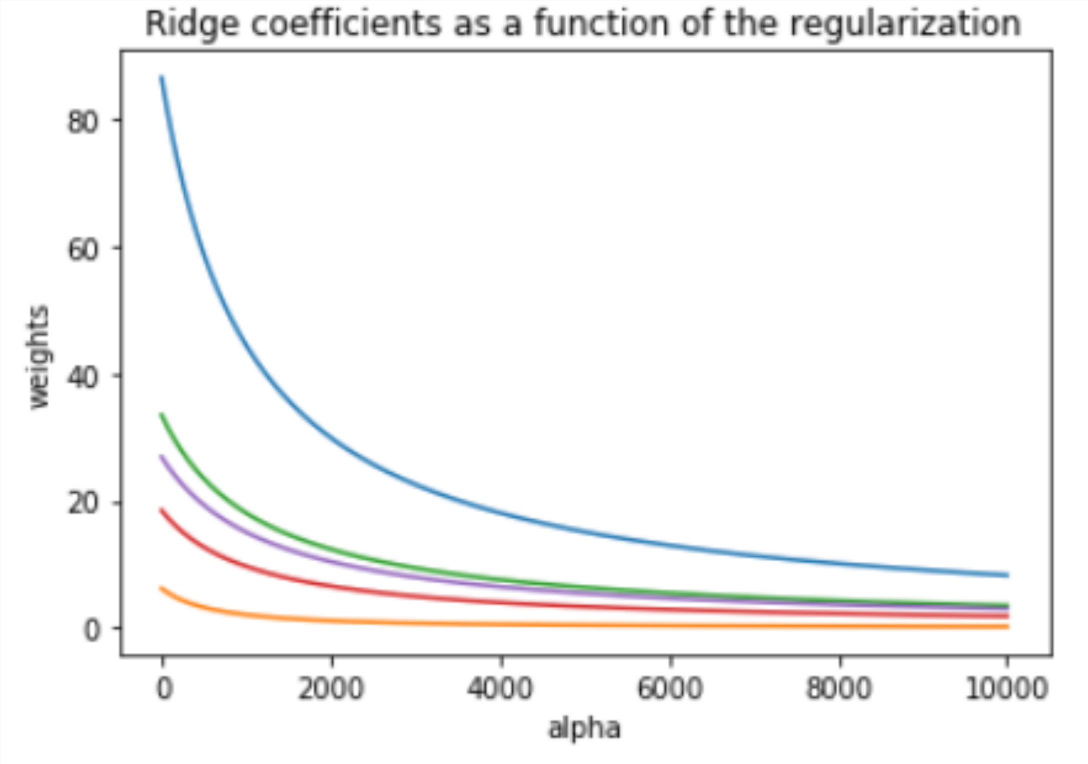




Ridge visualized

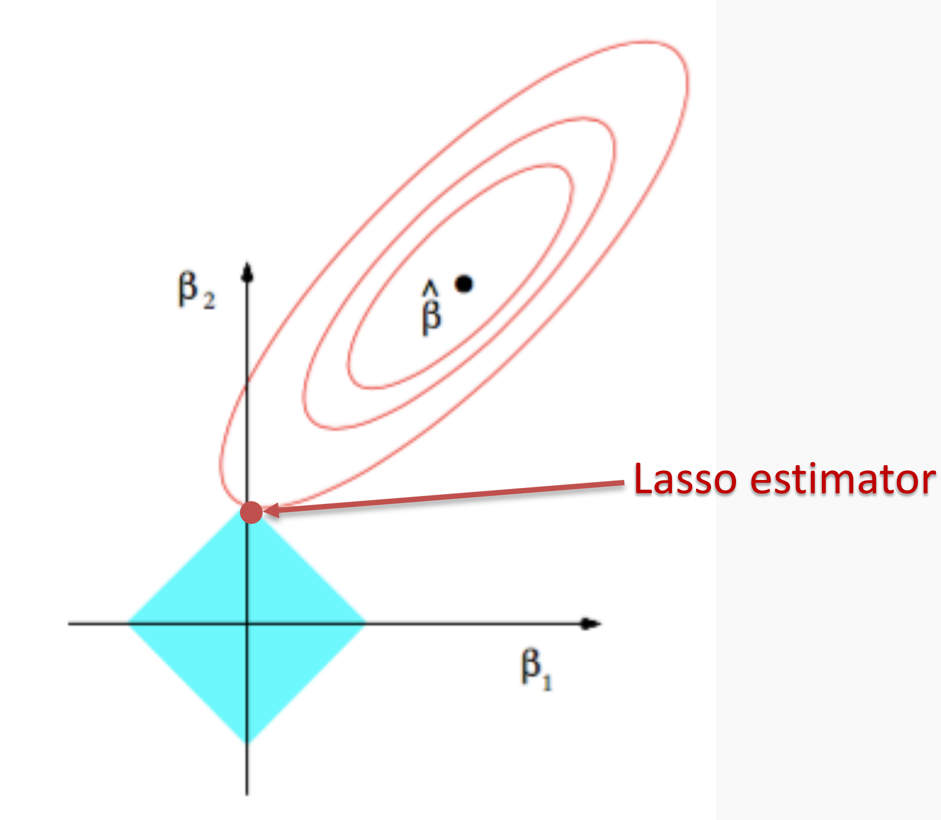


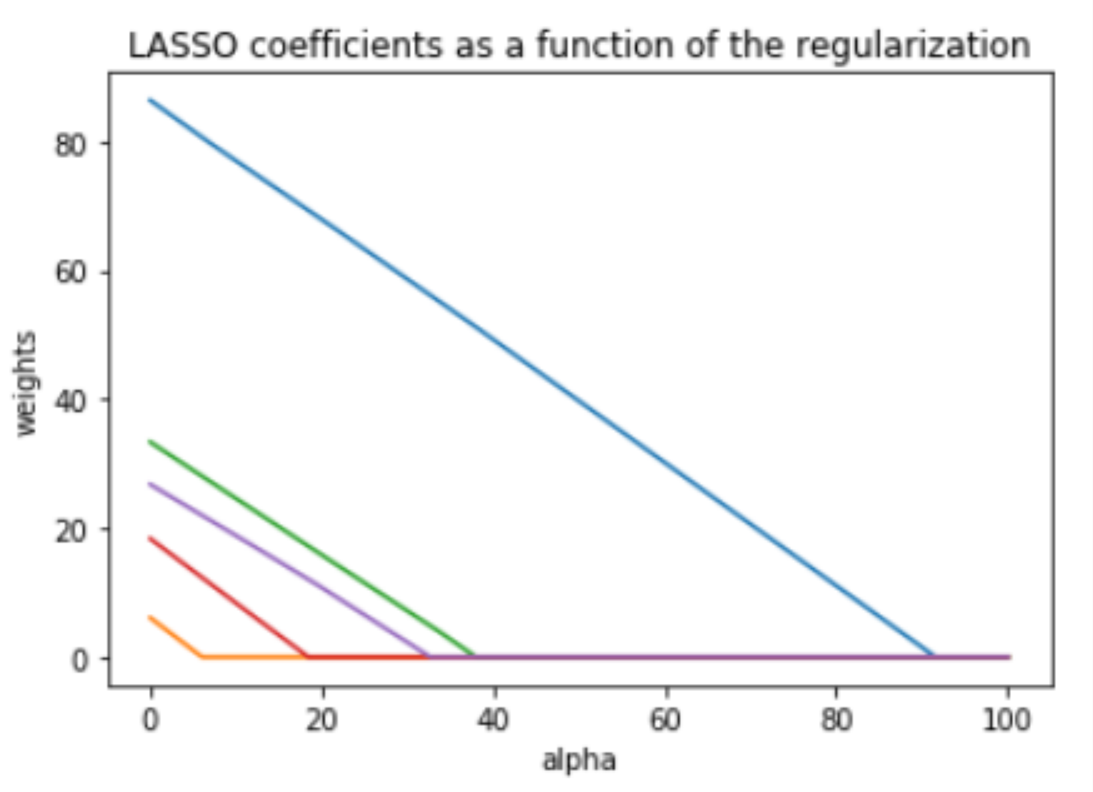
The ridge estimator is where the constraint and the loss intersect.



The values of the coefficients decrease as lambda increases, but they are not nullified.

LASSO visualized





The Lasso estimator tends to zero out parameters as the OLS loss can easily intersect with the constraint on one of the axis.

The values of the coefficients decrease as lambda increases, and are nullified fast.

Variable Selection as Regularization

Since LASSO regression tend to produce zero estimates for a number of model parameters - we say that LASSO solutions are **sparse** - we consider LASSO to be a method for variable selection.

Many prefer using LASSO for variable selection (as well as for suppressing extreme parameter values) rather than stepwise selection, as LASSO avoids the statistic problems that arises in stepwise selection.

Question: What are the pros and cons of the two approaches?