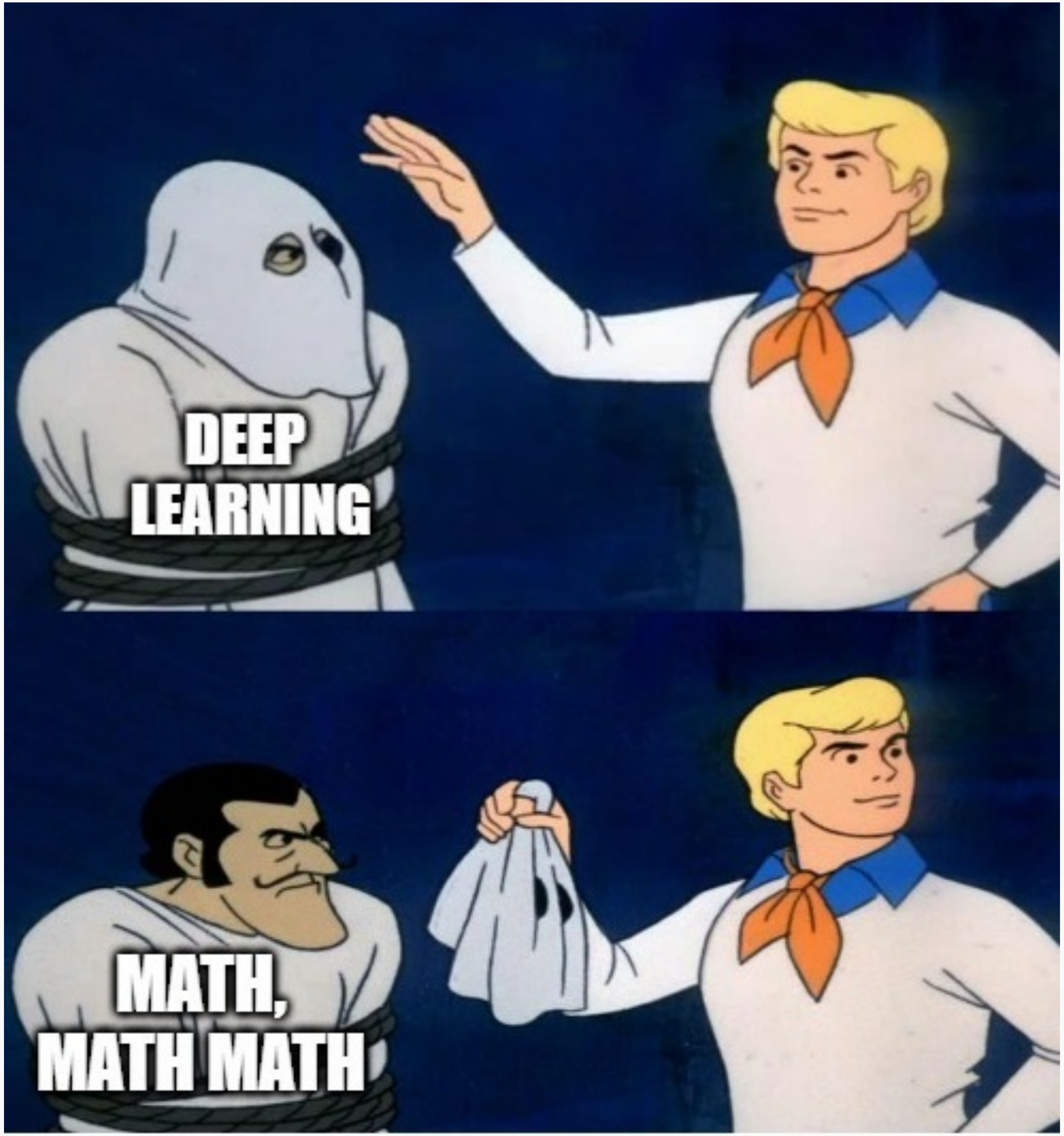
Part A: Universal Approximators; Nodes and Layers

CS109A Introduction to Data Science

Pavlos Protopapas, Kevin Rader and Chris Tanner


DEEP
LEARNING
MATH,
MATH MATH

Design Choices

Activation function

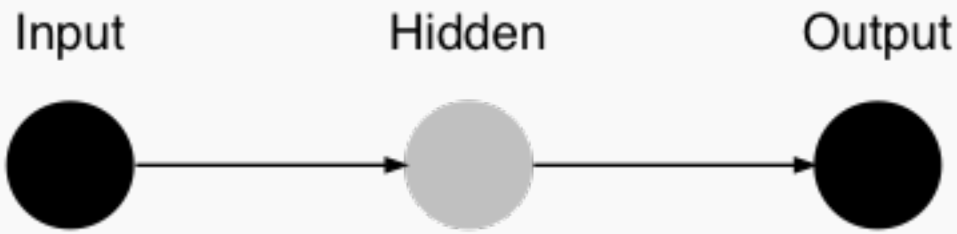
Loss function

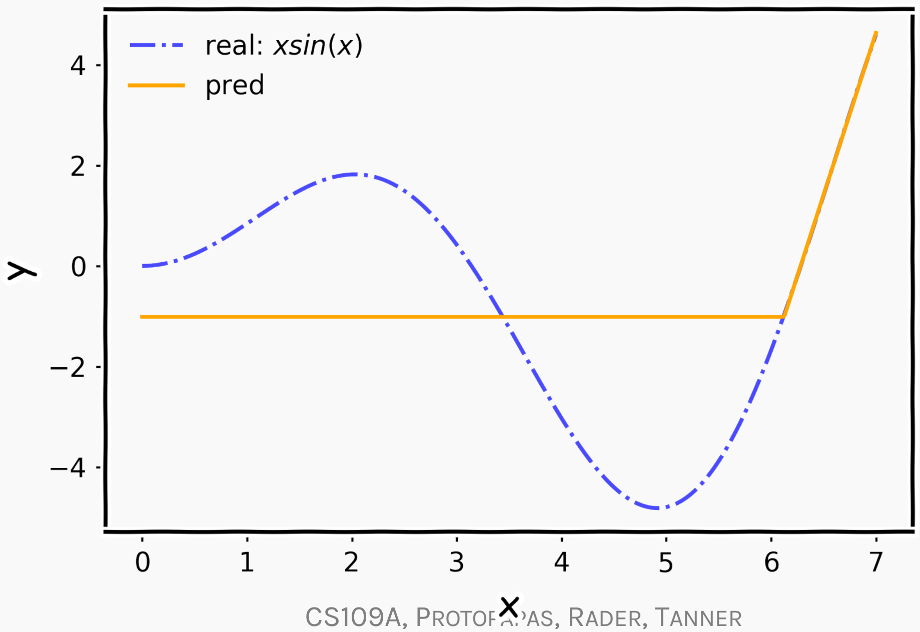
Output units

Architecture

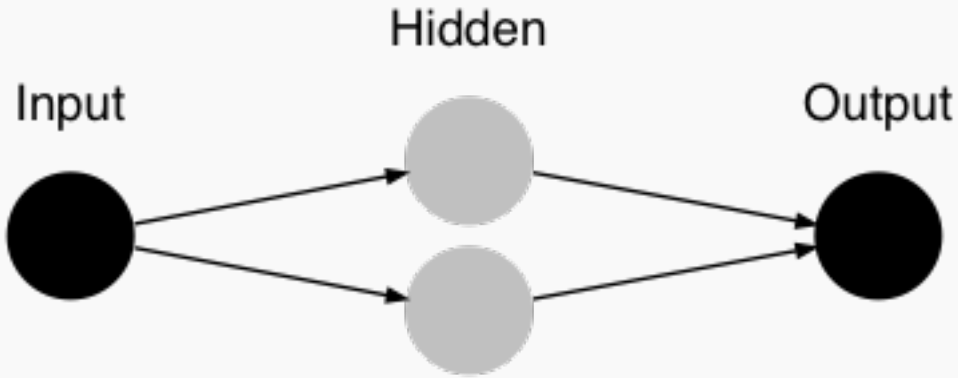
Optimizer

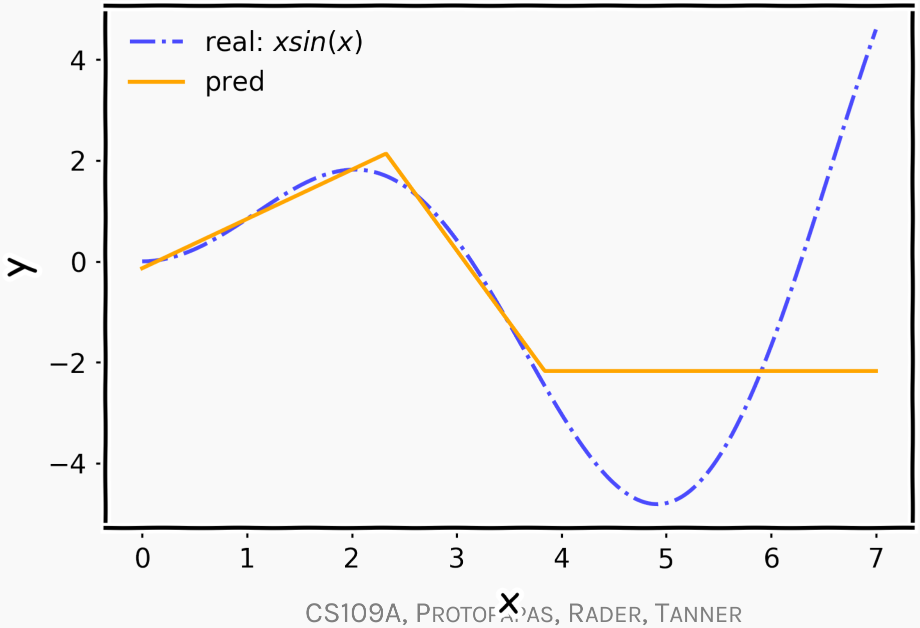
Number of nodes



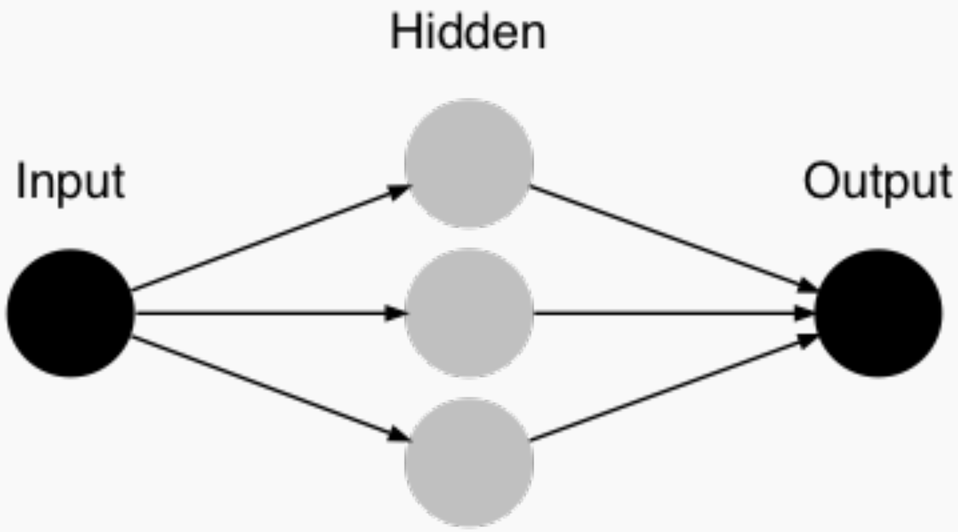


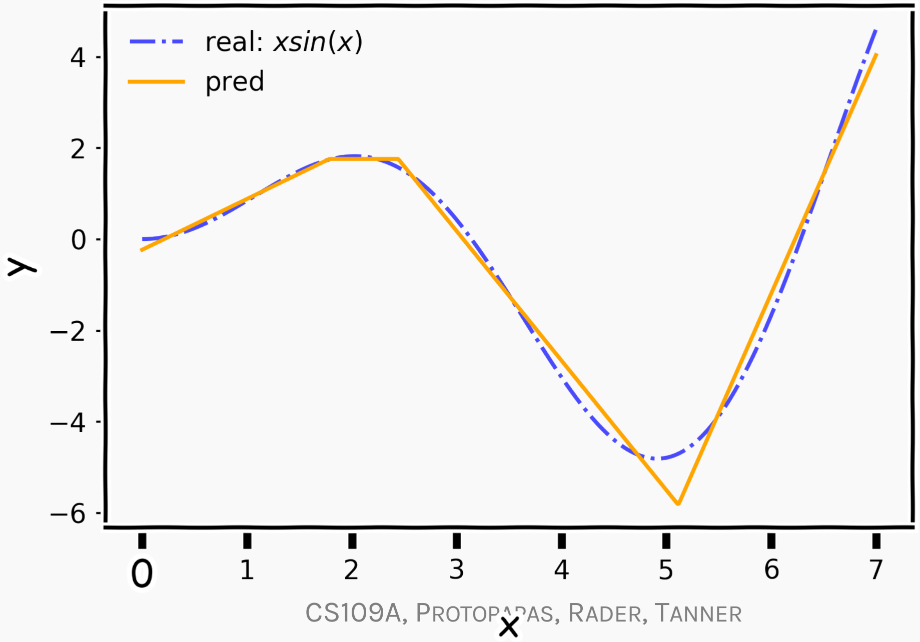
Number of nodes



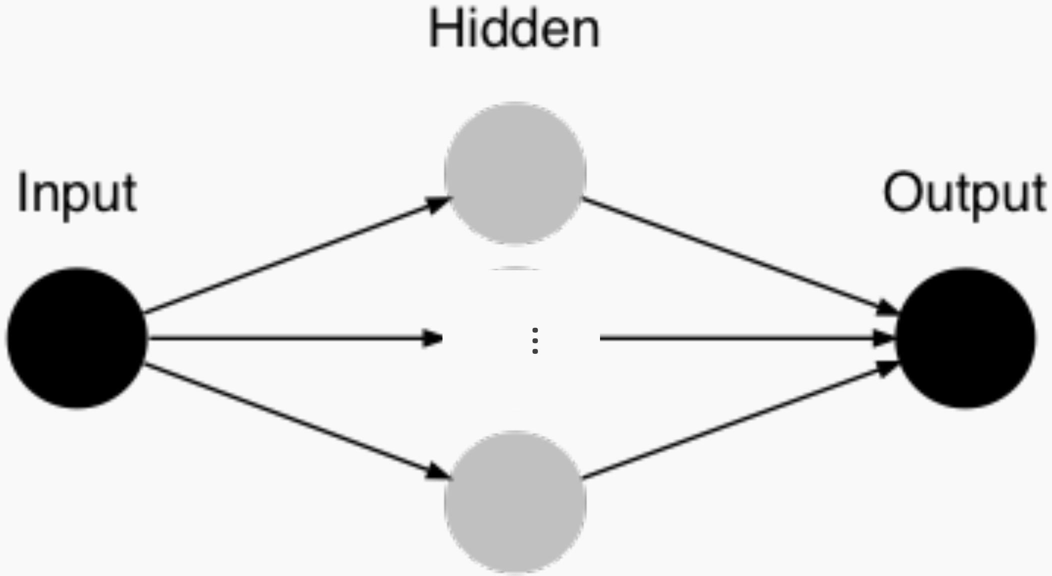


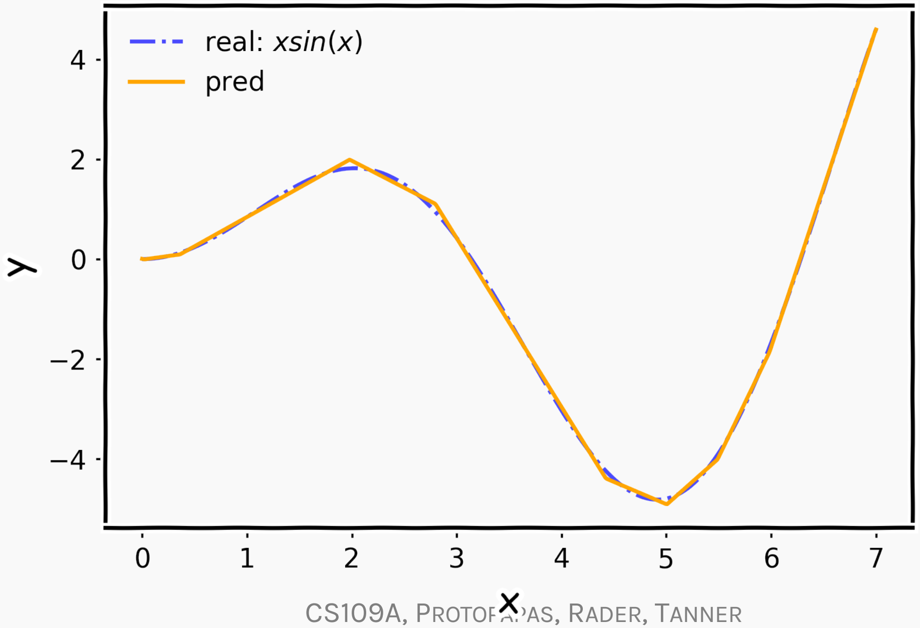
Number of nodes



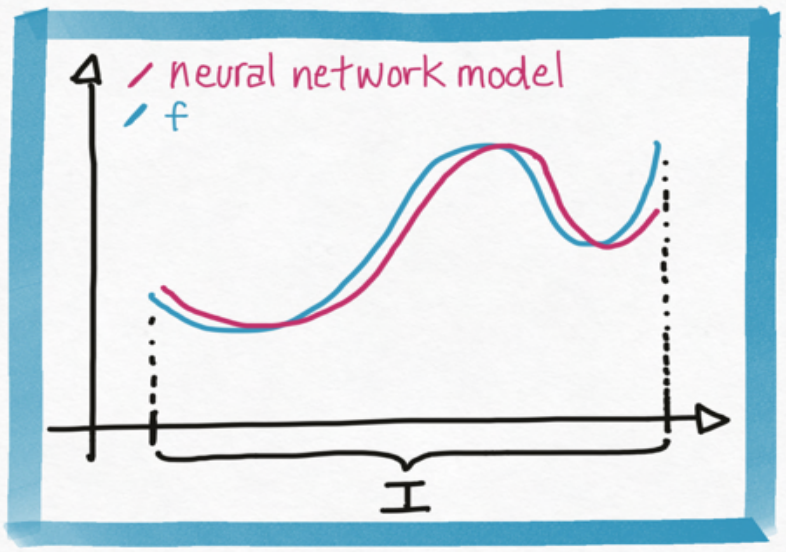


Number of nodes





Neural Networks as Universal Approximators



We have seen that neural networks can represent complex functions, but are there limitations on what a neural network can express?

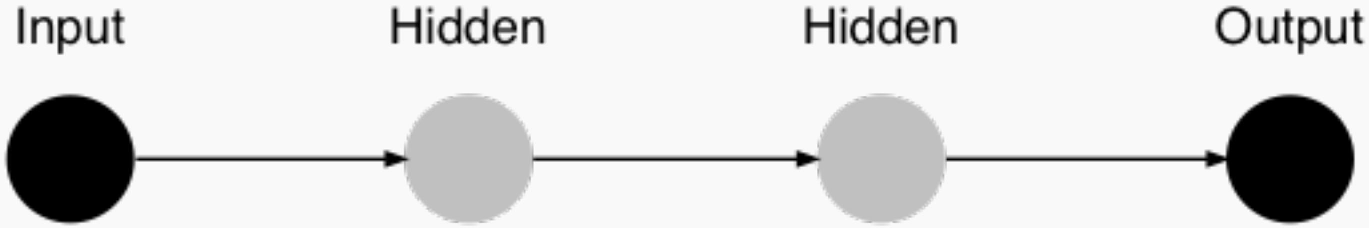
Theorem:

For any continuous function f defined on a bounded domain, we can find a neural network that approximates f with an arbitrary degree of accuracy.

One hidden layer is enough to represent an approximation of any function to an arbitrary degree of accuracy.

So why deeper?

Layers


Input
Hidden
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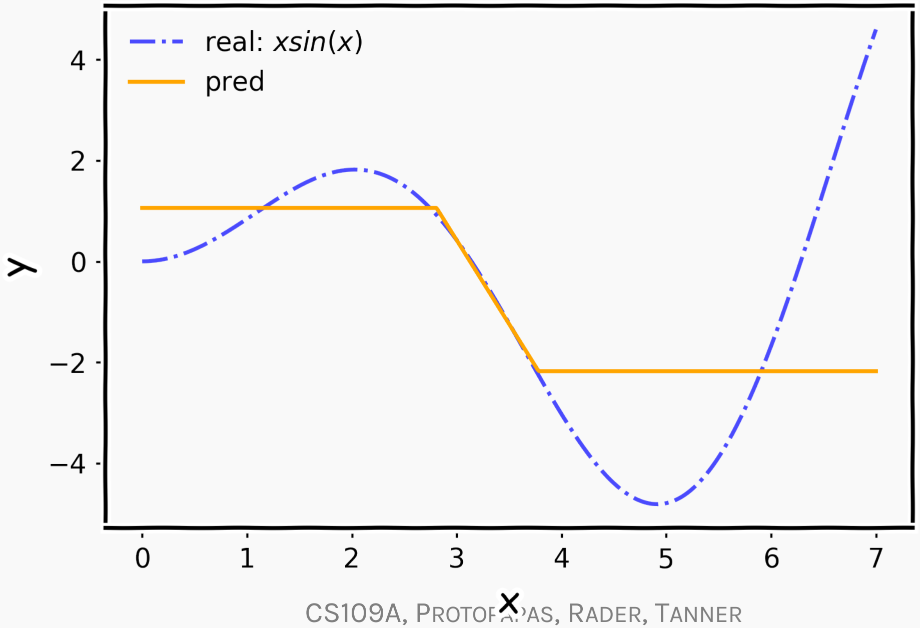
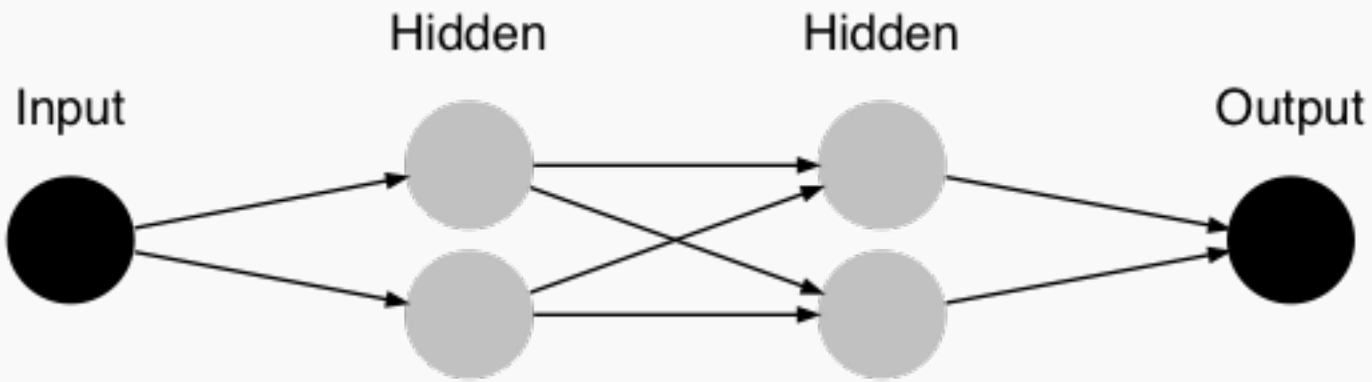
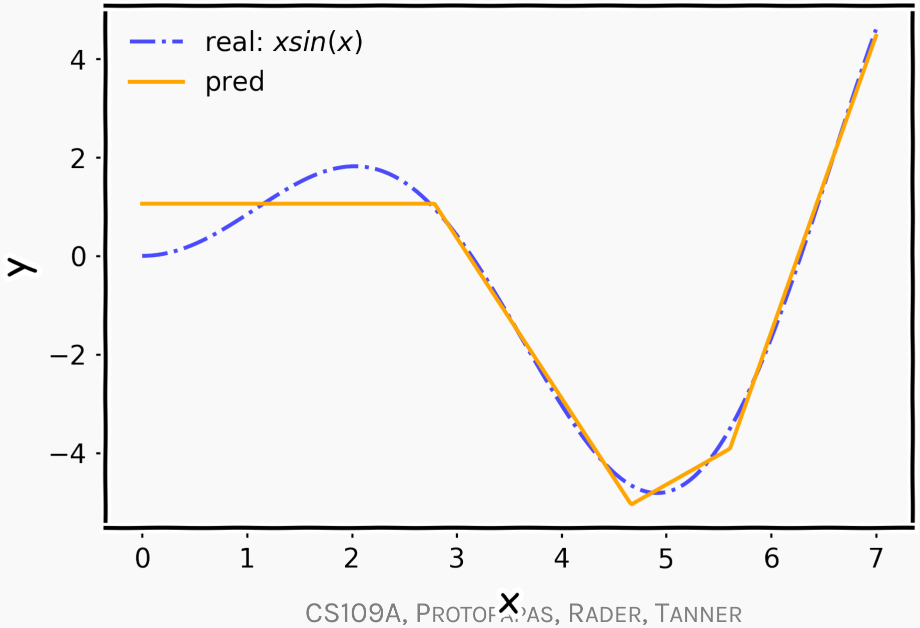
Output


real: $xsin(x)$
pred
y
x
4
2
0
−2
−4
0
1
2
3
4
5
6
7

Layers


Input
Hidden
Hidden
Output


real: $xsin(x)$
pred
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4
2
0
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2
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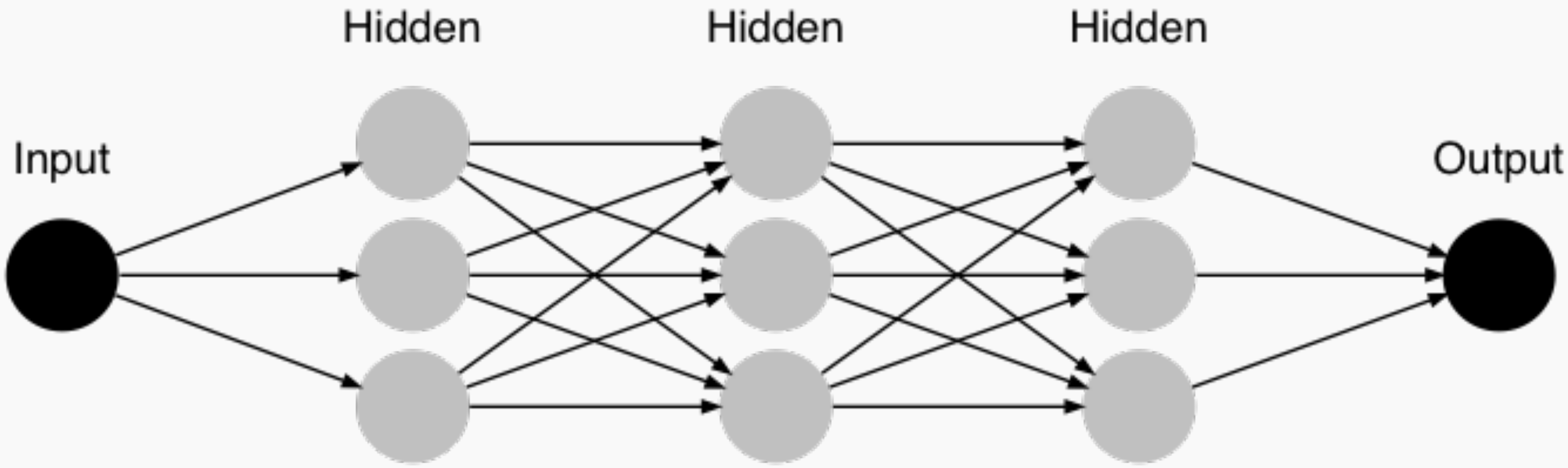
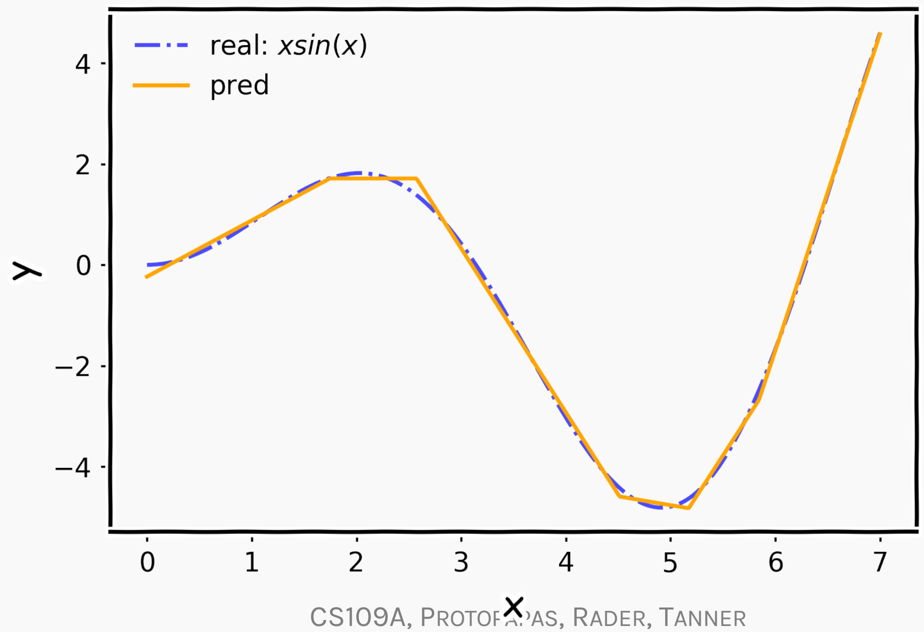
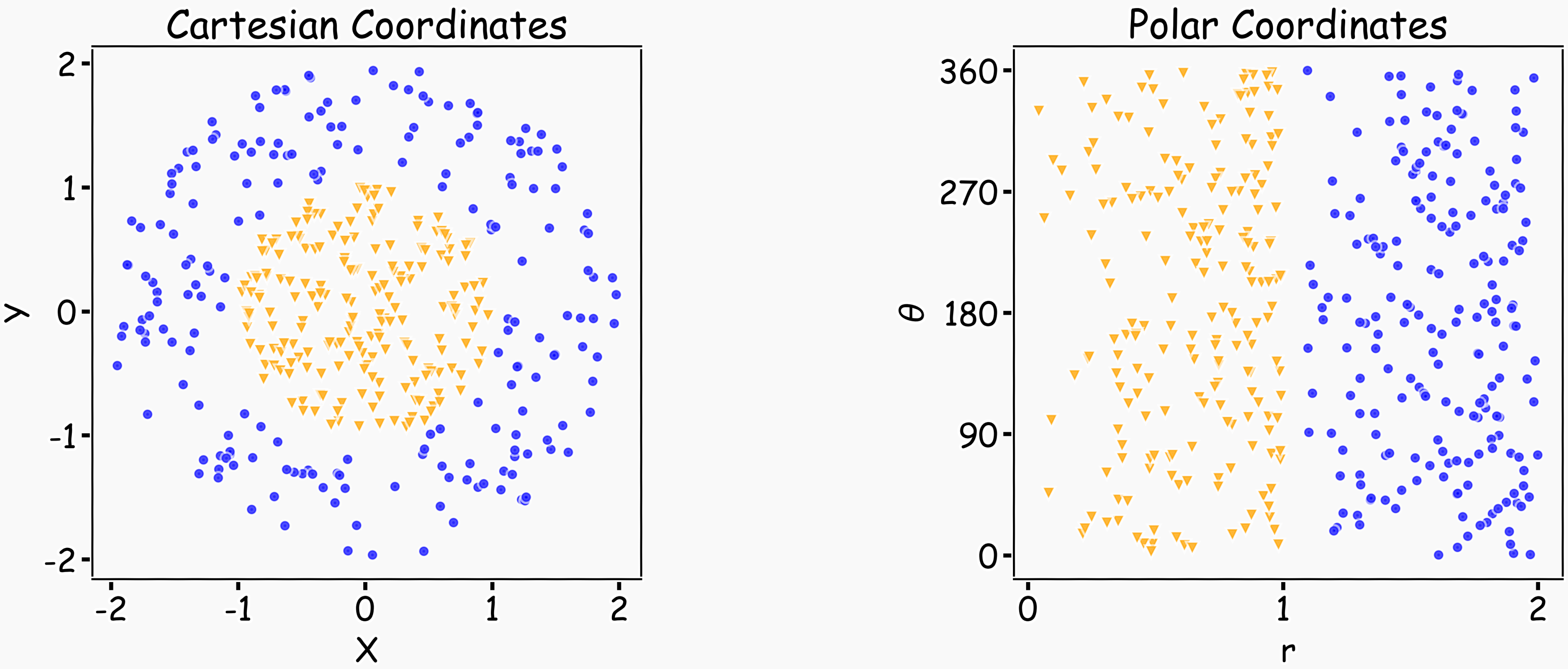
Layers


Hidden
Hidden
Hidden
Input
Output

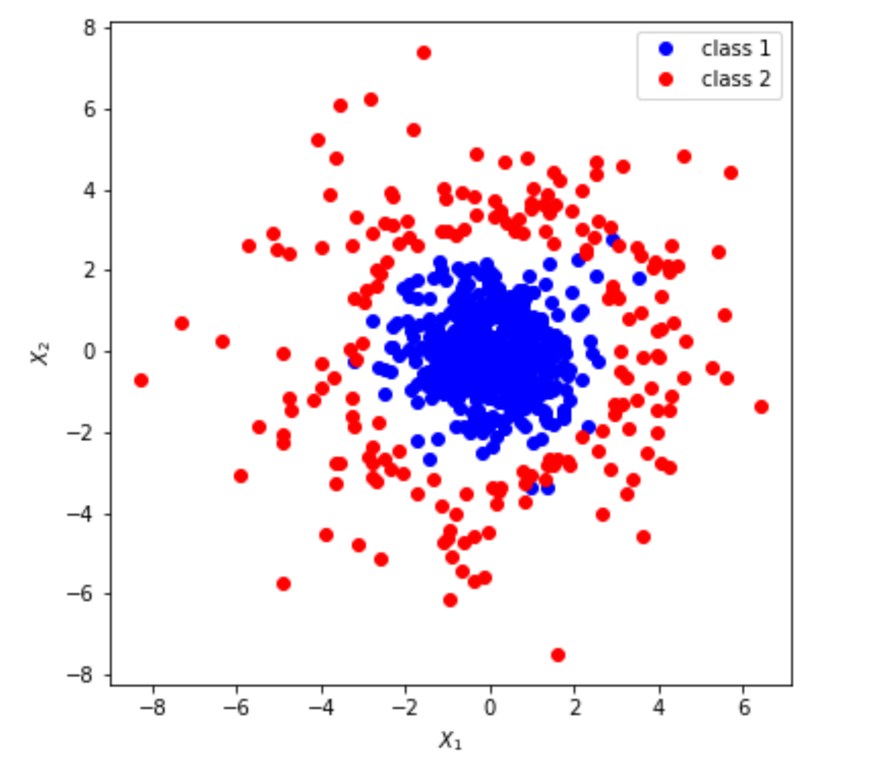

real: $xsin(x)$
pred
4
2
0
−2
−4
y
0
1
2
3
4
5
6
7
x

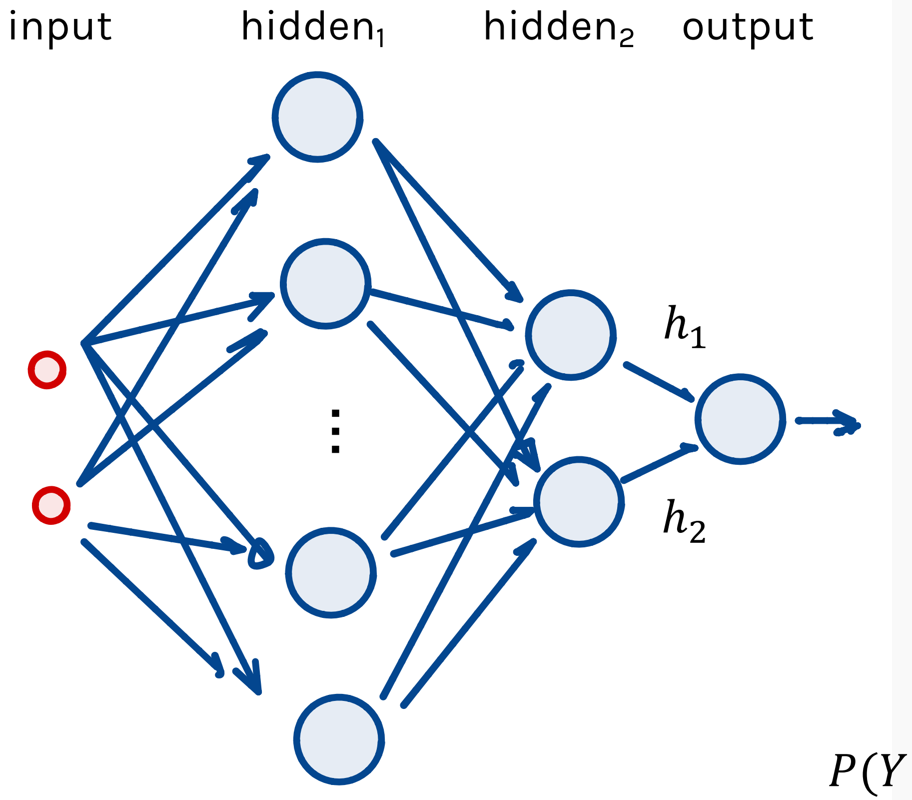
Why layers?

Representation matters!



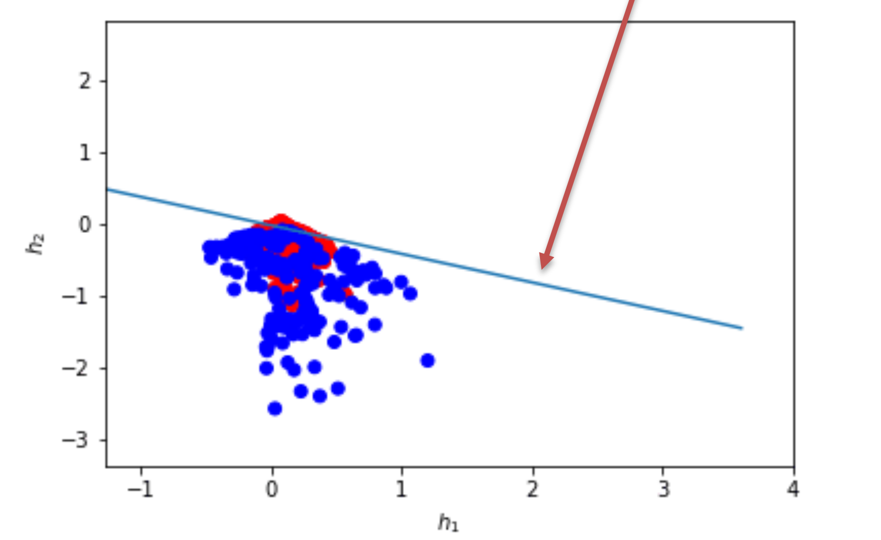
Neural networks can **learn useful representations** for the problem. This is another reason why they can be so powerful!



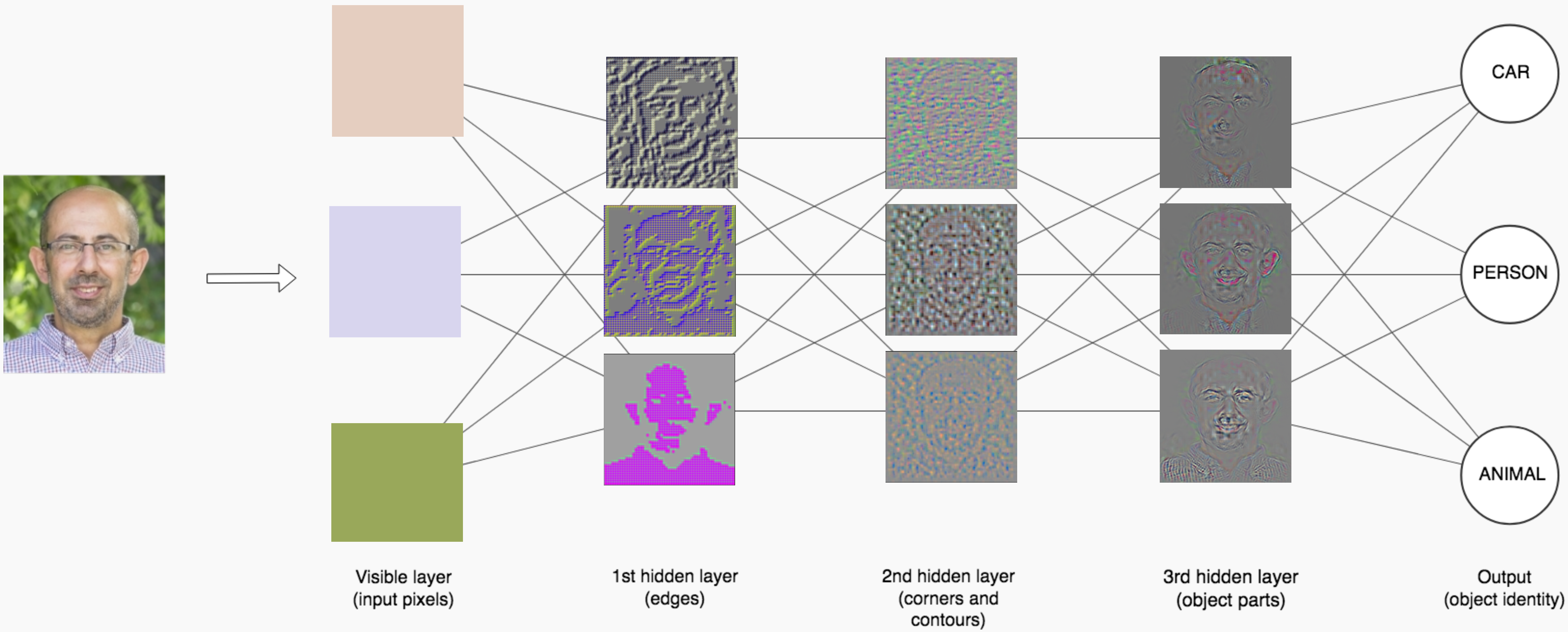


$P(Y = 1)$

$P(Y = 0.5)$



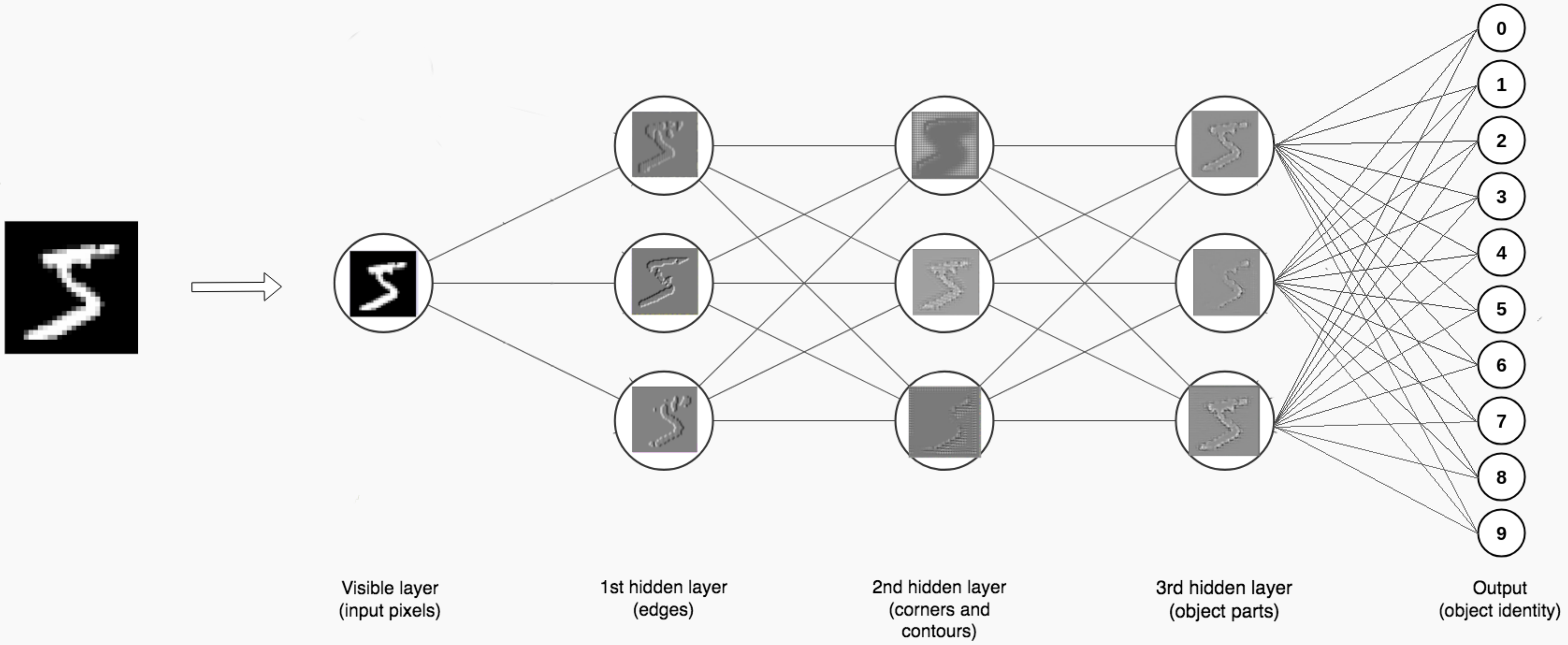
Depth = Repeated Compositions



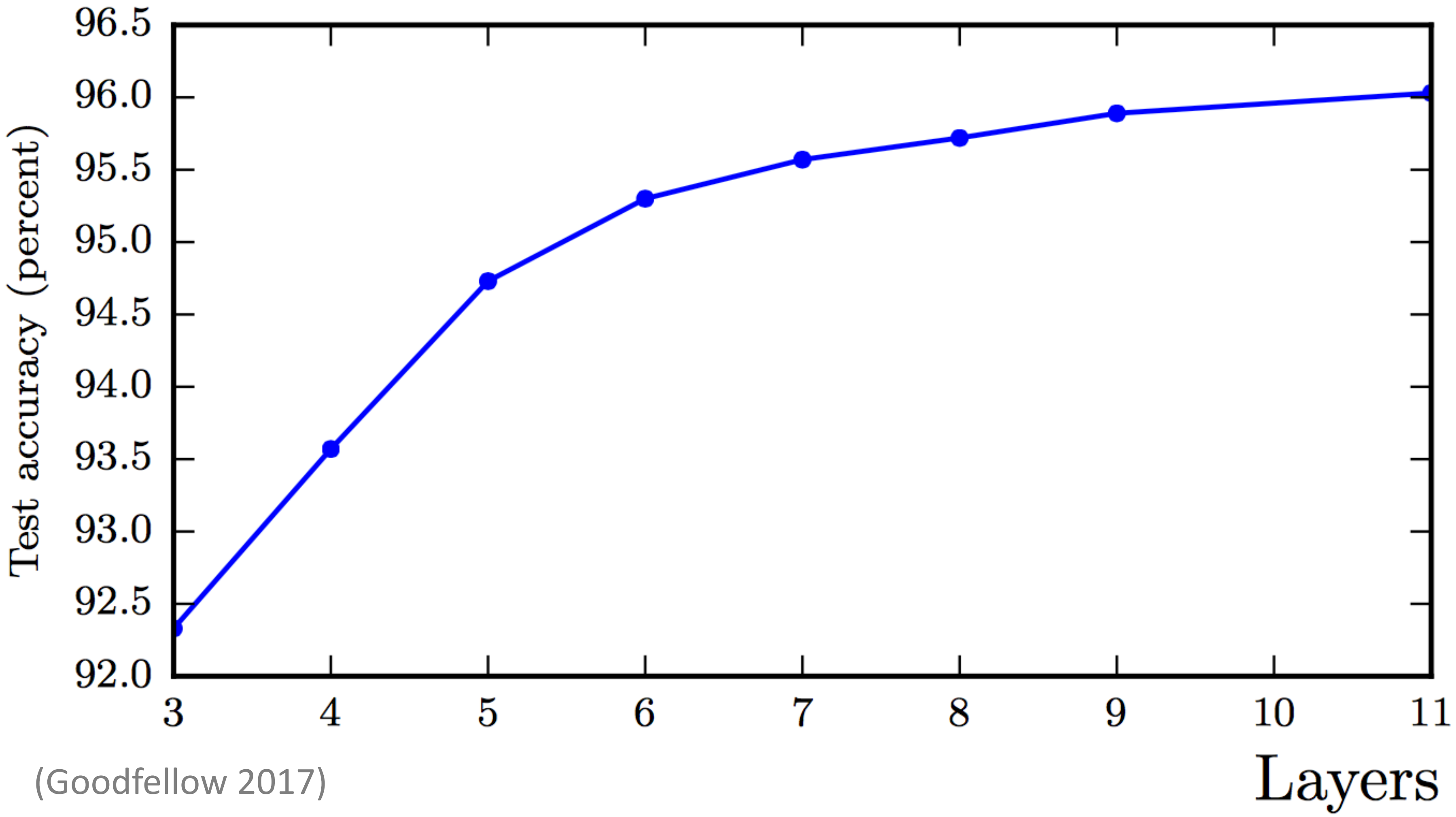
Neural Networks

Hand-written digit recognition: MNIST data

Depth = Repeated Compositions



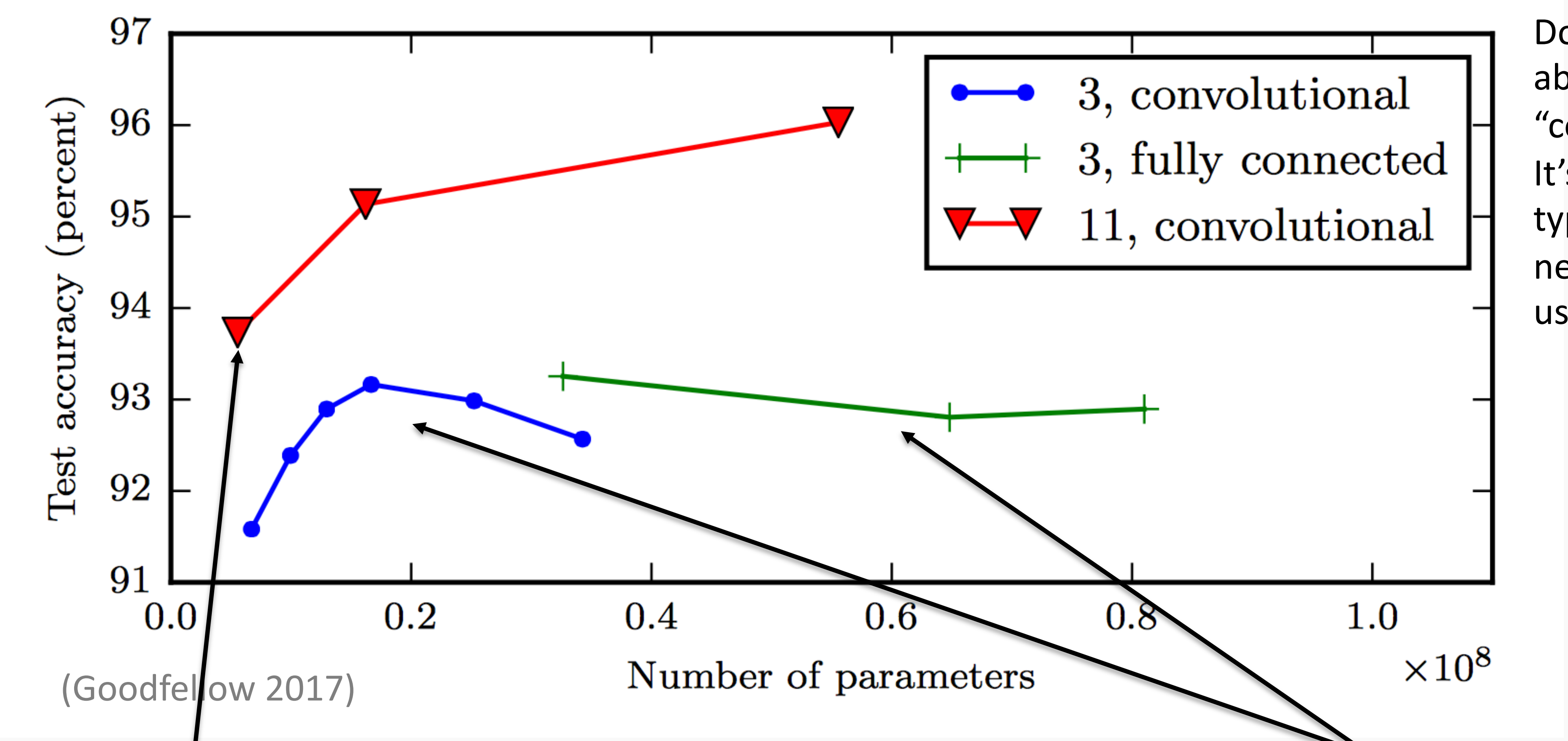
Better Generalization with Depth



(Goodfellow 2017)

Shallow Nets Overfit More

Depth helps, and it's not just because of more parameters

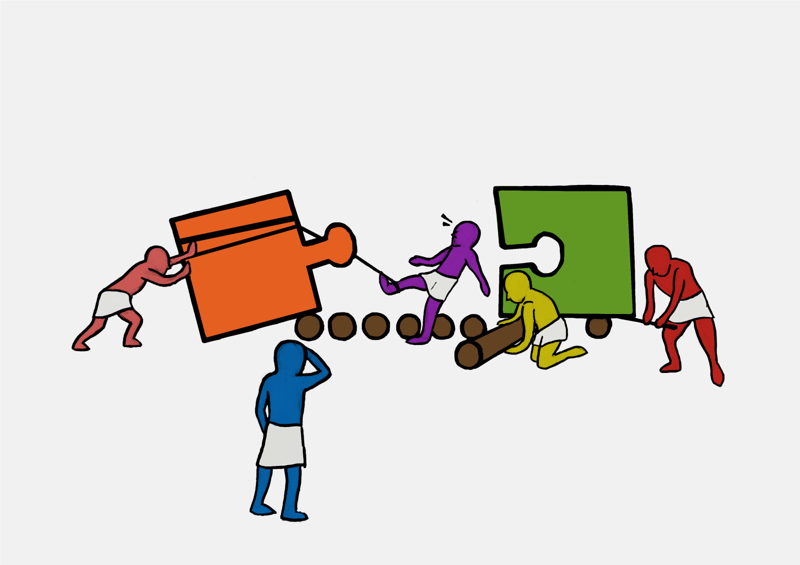


(Goodfellow 2017)

Don't worry about this word "convolutional". It's just a special type of neural network, often used for images.

The **11-layer net** generalizes better on the test set when controlling for number of parameters.

The 3-layer nets perform worse on the test set, even with similar number of total parameters.



Classifier using Keras on Iris data