Introduction to Regression

Part B: Error Evaluation and Model Comparison

CS109A Introduction to Data Science

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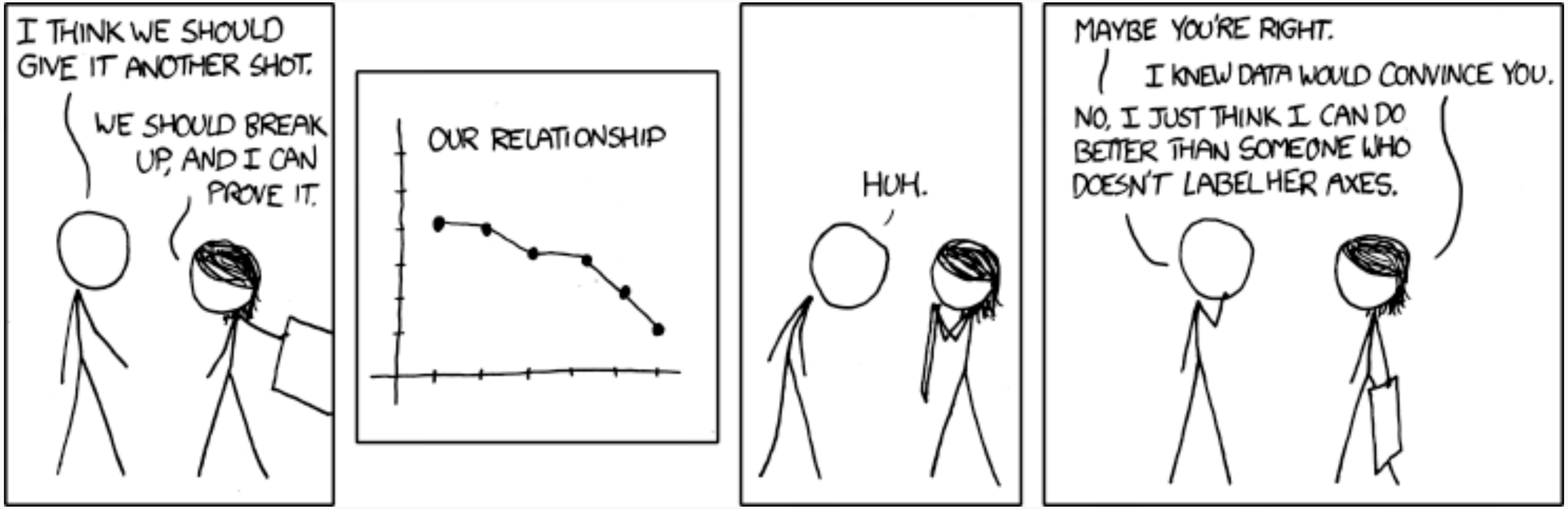
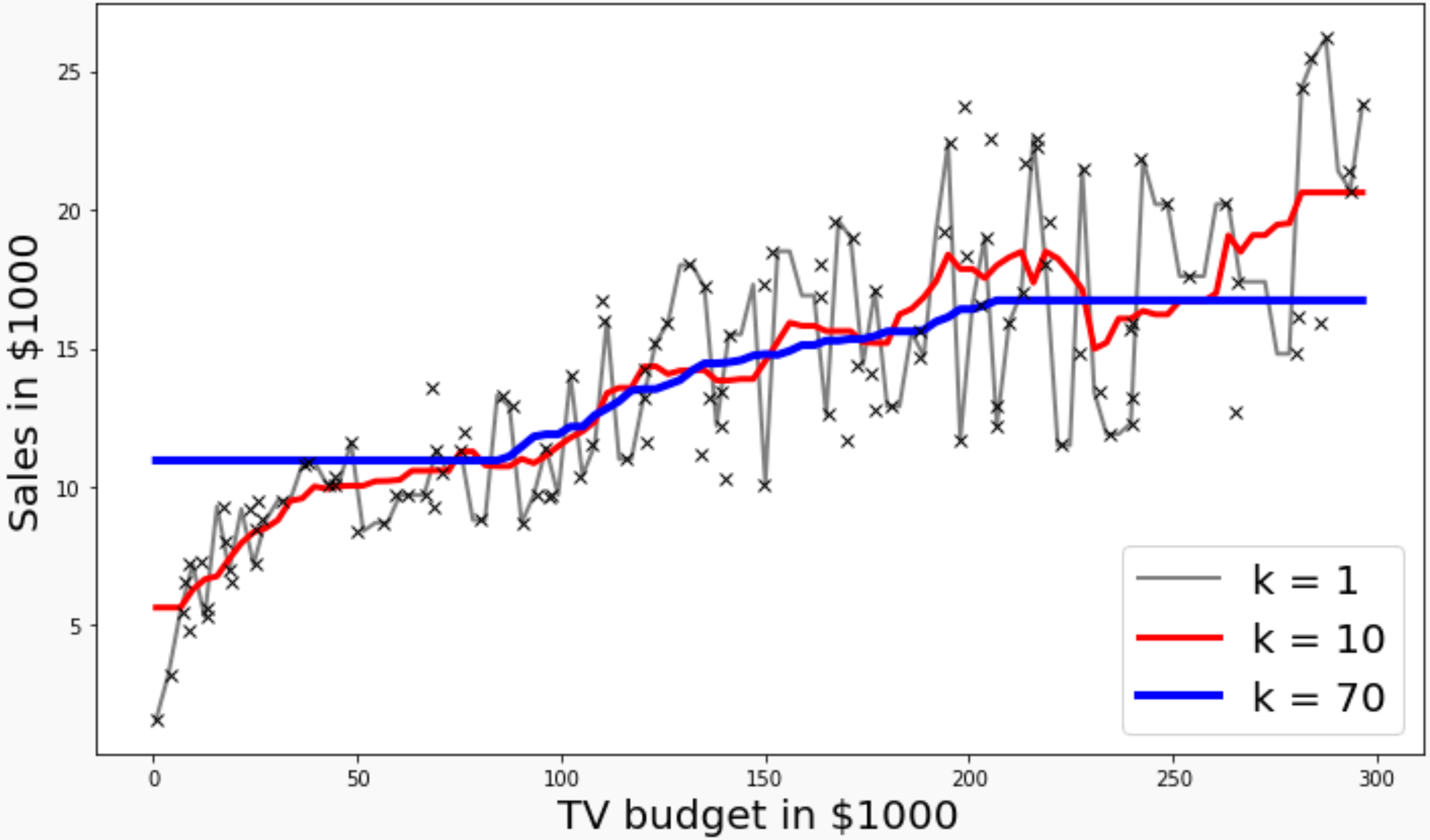

I THINK WE SHOULD GIVE IT ANOTHER SHOT.
WE SHOULD BREAK UP, AND I CAN PROVE IT.
OUR RELATIONSHIP
HUH.
MAYBE YOU'RE RIGHT.
I KNEW DATA WOULD CONVINCE YOU.
NO, I JUST THINK I CAN DO BETTER THAN SOMEONE WHO DOESN'T LABEL HER AXES.

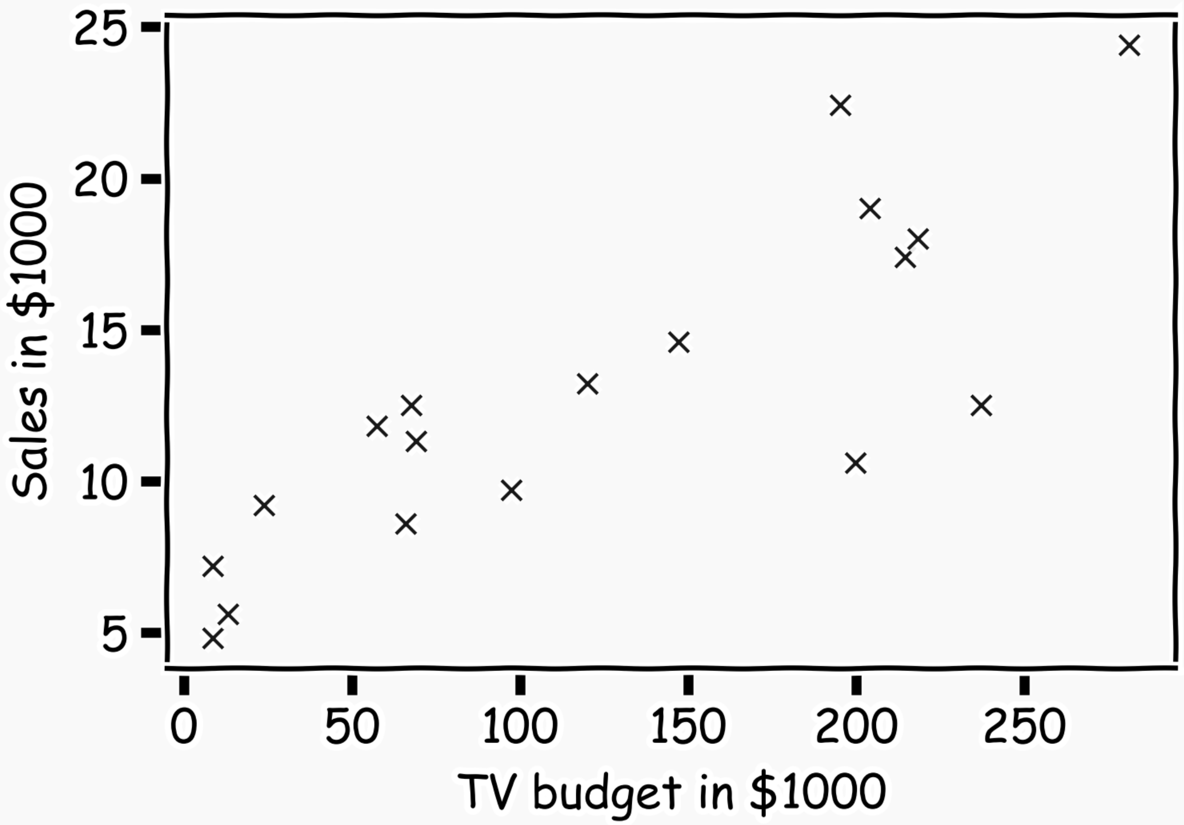


Sales in $1000
TV budget in $1000
25
20
15
10
5
0
50
100
150
200
250
300
k = 1
k = 10
k = 70

Error Evaluation

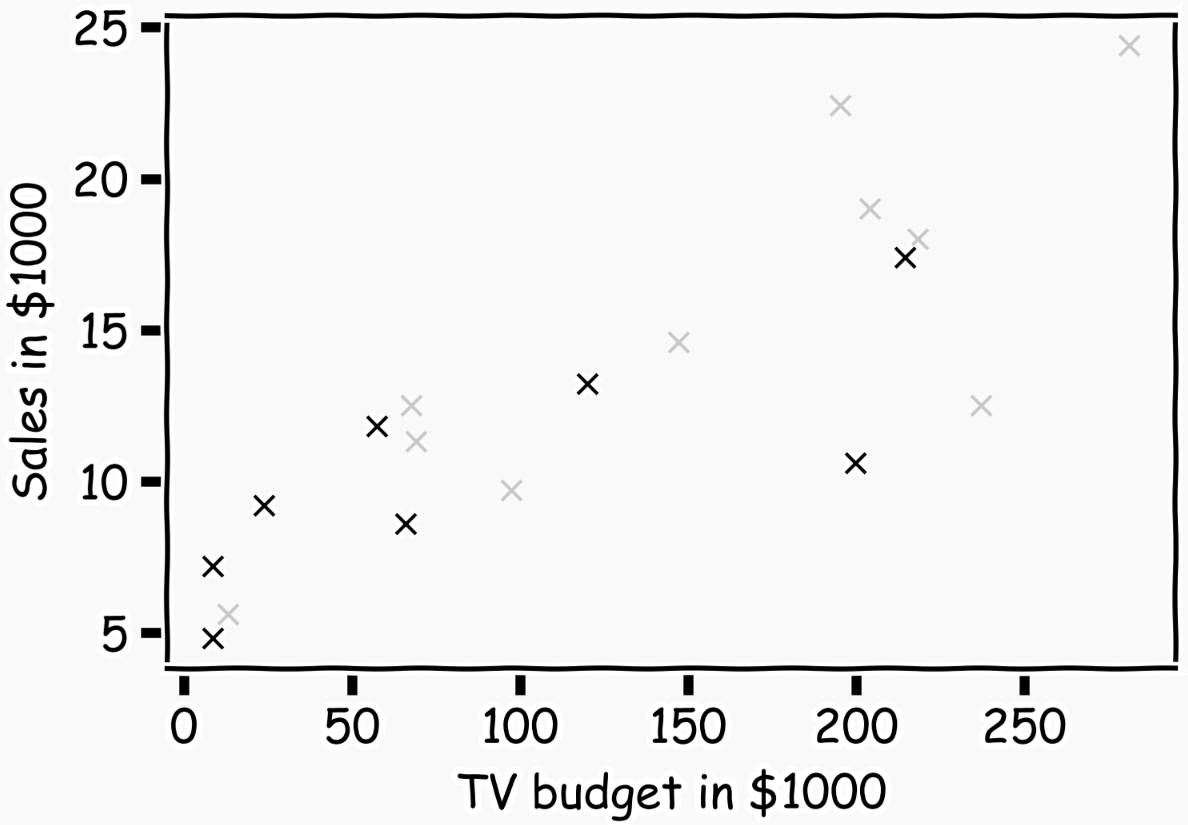
Error Evaluation

Start with some data.



Error Evaluation

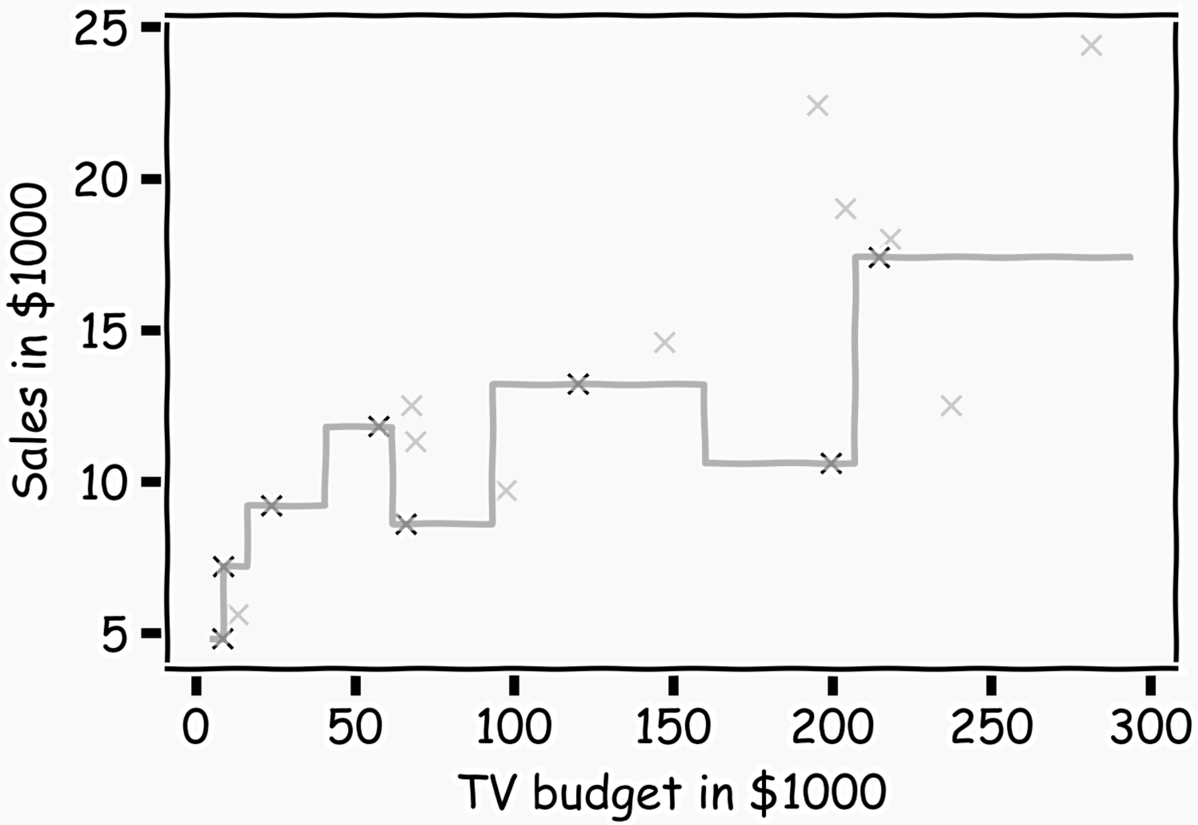
Hide some of the data from the model. This is called **train-test** split.



We use the train set to estimate $\hat{y}$, and the test set to evaluate the model.

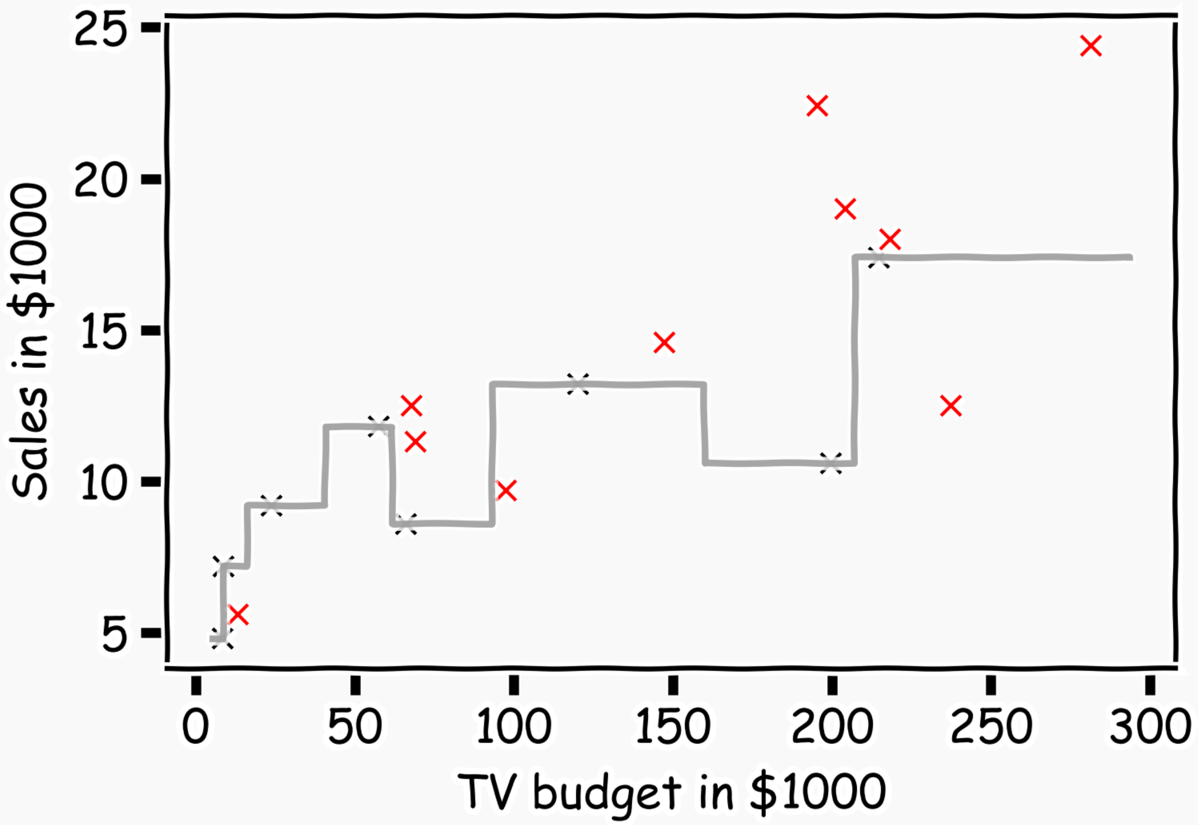
Error Evaluation

Estimate $\hat{y}$ for k=1 .



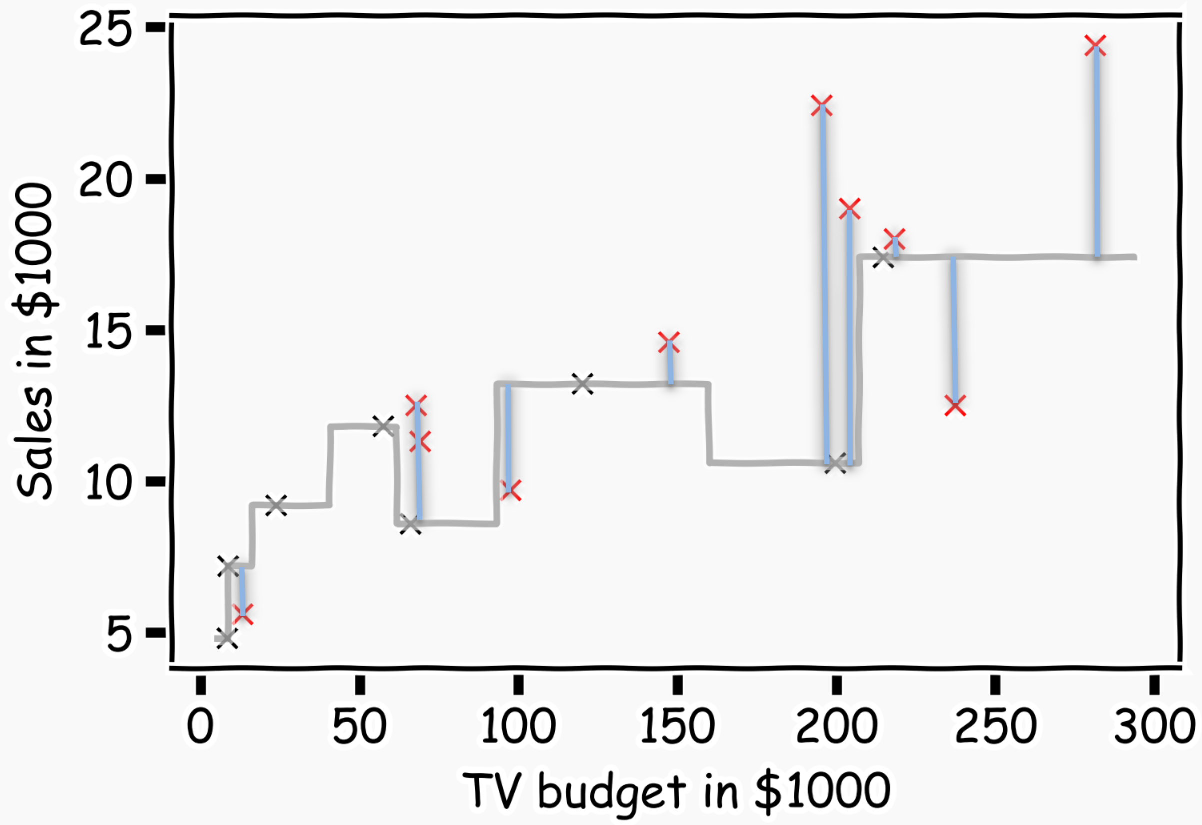
Error Evaluation

Now, we look at the data we have not used, the **test data** (red crosses).



Error Evaluation

Calculate the **residuals** $(y_i - \hat{y}_i)$.



Error Evaluation

In order to quantify how well a model performs, we aggregate the errors and we call that the ***loss*** or ***error*** or ***cost function.***

A common loss function for quantitative outcomes is the **Mean Squared Error (MSE):**

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2$$

Note: Loss and cost function refer to the same thing. Cost usually refers to the total loss where loss refers to a single training point.

Error Evaluation

Caution: The MSE is by no means the only valid (or the best) loss function!

1. Max Absolute Error
2. Mean Absolute Error
3. Mean Squared Error

We will motivate MSE when we introduce probabilistic modeling.

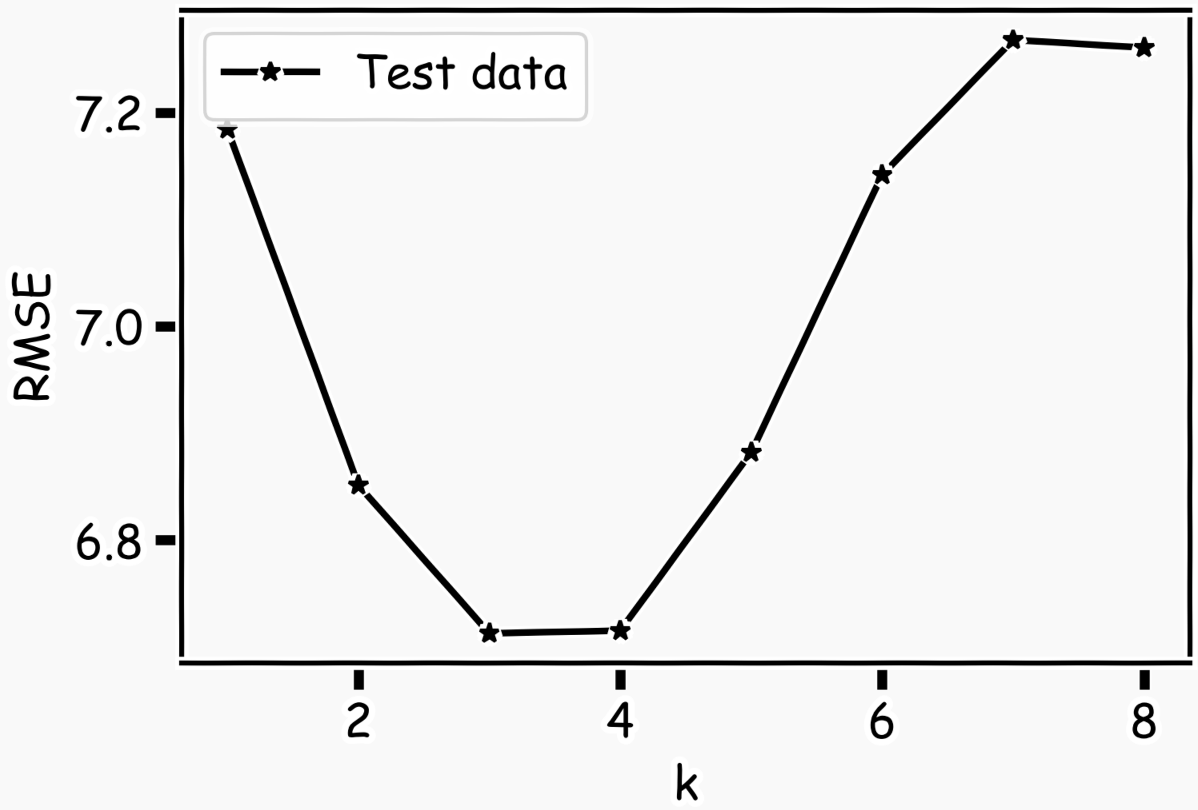
Note: The square **R**oot of the **M**ean of the **S**quared **E**rrors (RMSE) is also commonly used.

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y_i})^2}$$

Model Comparison

Model Comparison

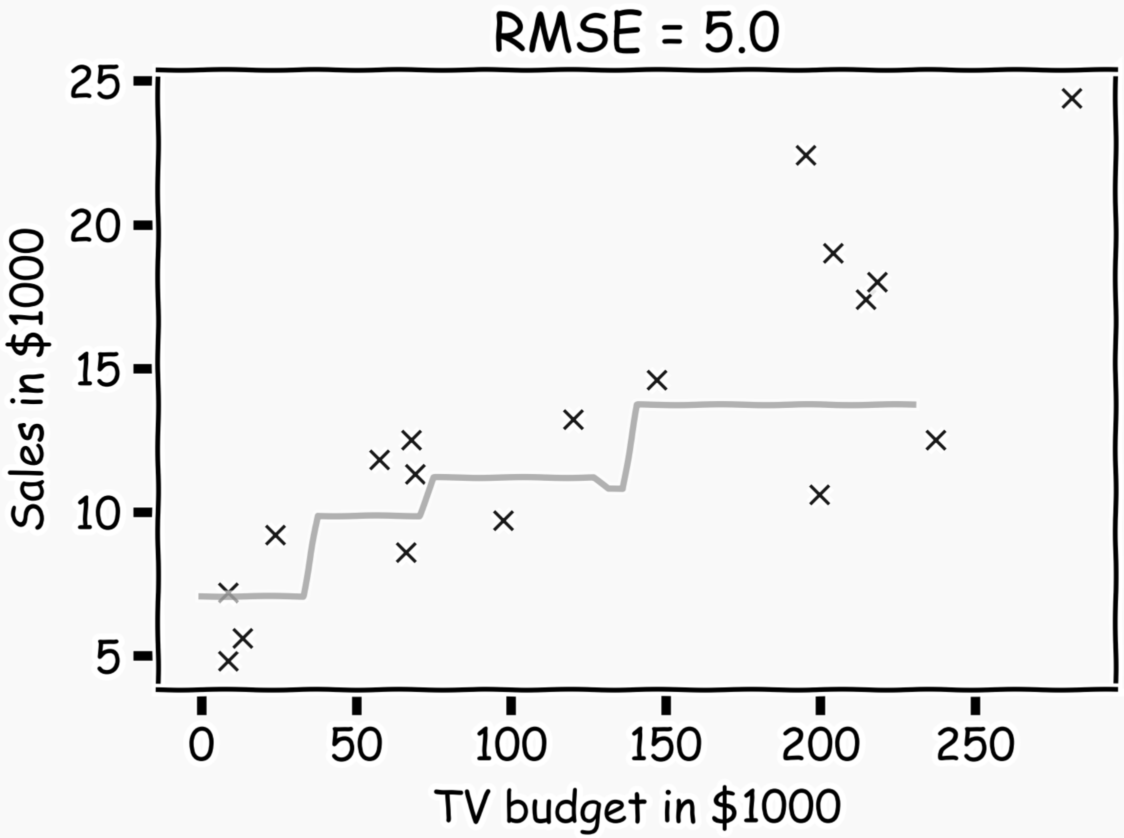
Do the same for all *k*'s and compare the RMSEs. *k*=3 seems to be the **best model**.



Model Fitness

Model fitness

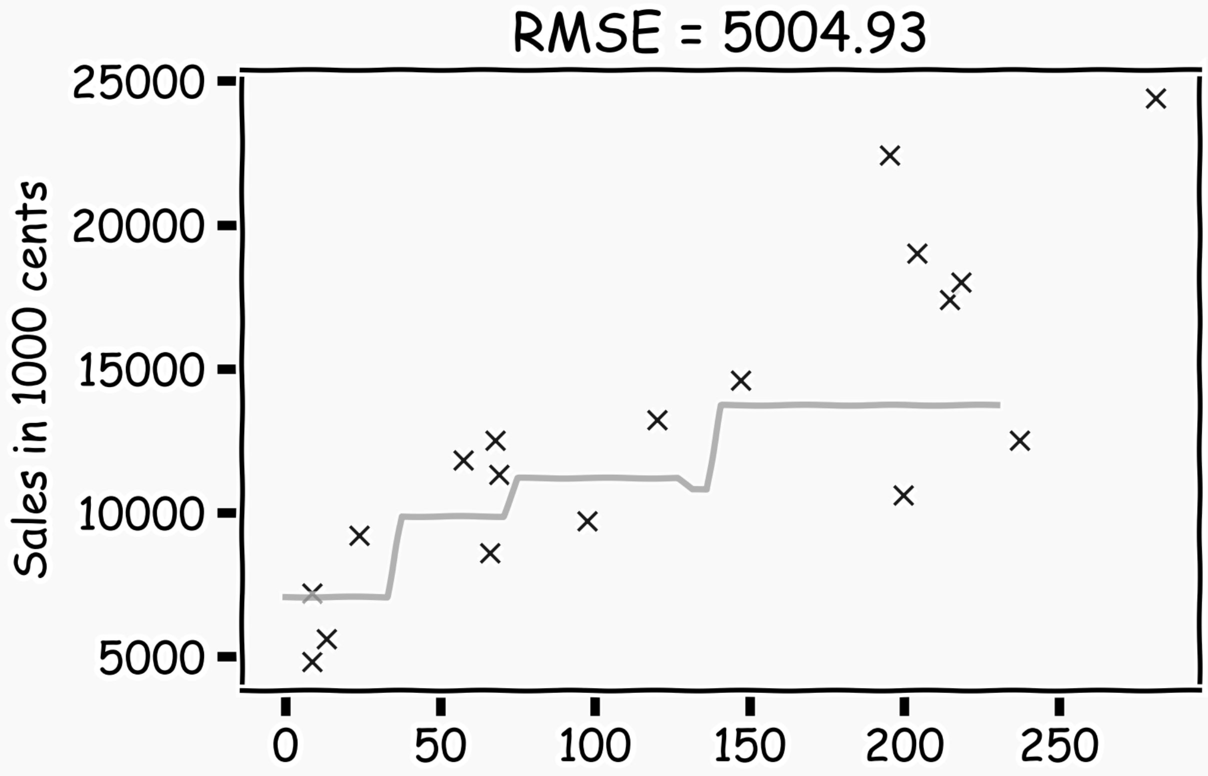
For a subset of the data, calculate the RMSE for *k*=3.



Is RMSE=5.0 good enough?

Model fitness

What if we measure the Sales in cents instead of dollars?

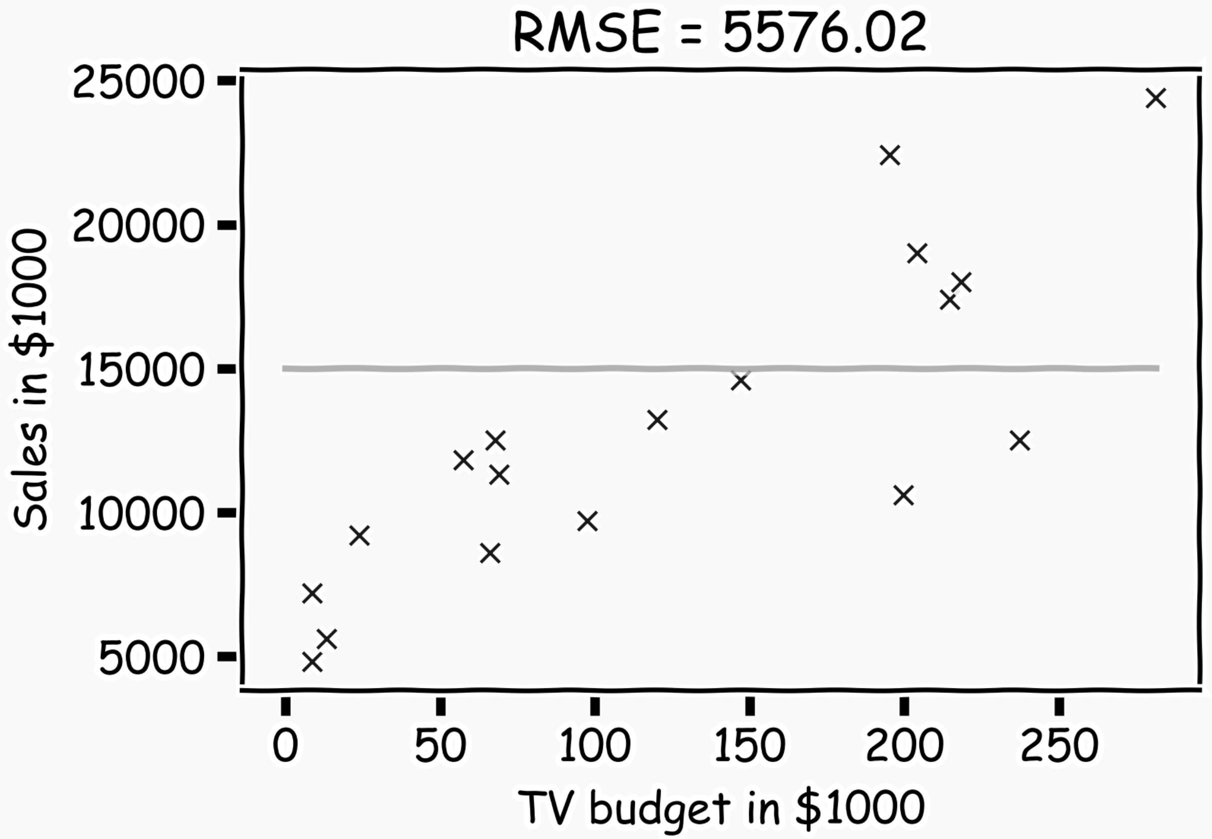


RMSE is now 5004.93.

Is that good?

Model fitness

It is better if we compare it to something.



We will use the simplest model:

$$\hat{y} = \bar{y} = \frac{1}{n}\sum_{i} y_i$$

as the worst possible model and

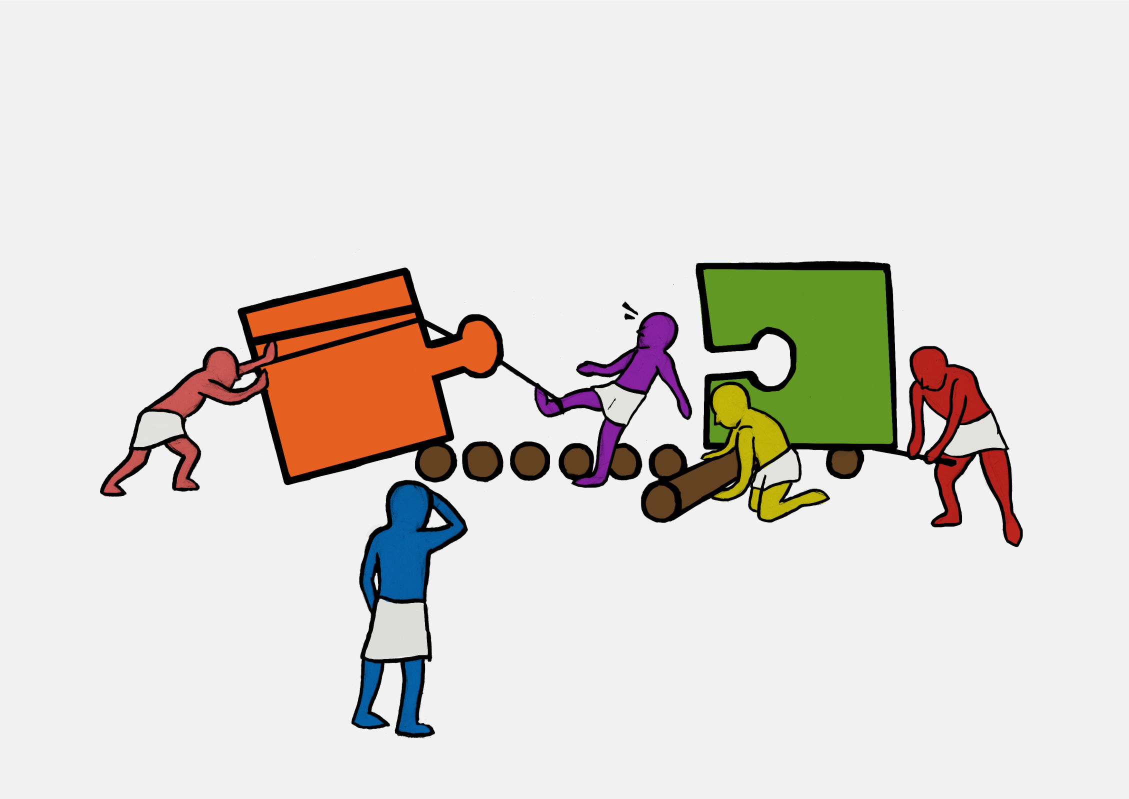
$$\hat{y_i} = y_i$$

as the best possible model.

R-squared

$$R^2 = 1 - \frac{\sum_i (\hat{y}_i - y_i)^2}{\sum_i (\bar{y} - y_i)^2}$$

- If our model is as good as the mean value, $\bar{y}$, then $R^2 = 0$
- If our model is perfect then $R^2 = 1$
- R^2 can be negative if the model is worst than the average. This can happen when we evaluate the model in the test set.



Ex B.1