Lecture 18: Perceptron and Multilayer Perceptron

CS109A Introduction to Data Science

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ANNOUNCEMENTS

- Homework 5 (209) due on Wednesday 11:59 pm, Nov 6
- Advanced Section on Trees is on Wednesday Nov 13
- Finally

Outline

1. Introduction to Artificial Neural Networks
2. Review of Classification and Logistic Regression
3. Single Neuron Network (‘Perceptron’)
4. Multi-Layer Perceptron (MLP)

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1. **Introduction to Artificial Neural Networks**
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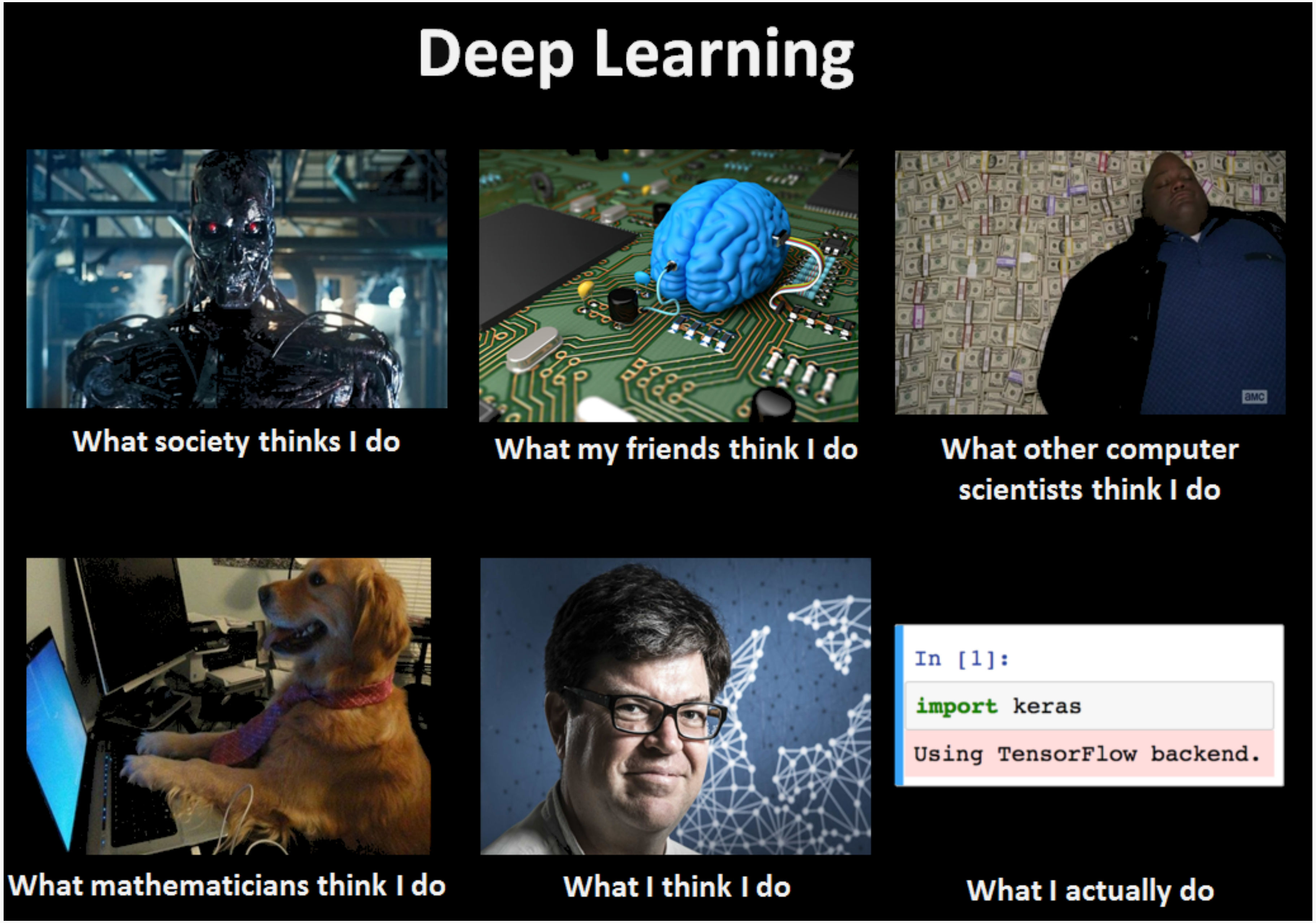
Artificial Neural Networks


Deep Learning
What society thinks I do
What my friends think I do
What other computer scientists think I do
What mathematicians think I do
What I think I do
In [1]:
import keras
Using TensorFlow backend.
What I actually do

Watch this!

http://video.arstechnica.com/watch/sunspring-sci-fi-short-film

Today's news

An AI just beat top lawyers at their own game

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IMAGE: BOB AL-GREEN/MASHABLE

BY MONICA CHIN

FEB 2018

The nation's top lawyers recently battled artificial intelligence in a competition to interpret contracts — and they lost.

A new study, conducted by legal AI platform LawGeex in consultati Share Quote ssors from Stanford University, Duke University School of Law, and University of Southern California, pitted twenty experienced lawyers against an AI trained to evaluate legal contracts.

Competitors were given four hours to review five non-disclosure agreements (NDAs) and identify 30 legal issues, including arbitration, confidentiality of relationship, and indemnification. They were scored by how accurately they identified each issue.

SEE ALSO: Google's new AI can predict heart disease by simply scanning your eyes

Today's news

Google's new AI can predict heart disease by simply scanning your eyes

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IMAGE: BEN BRAIN/DIGITAL CAMERA MAGAZINE VIA GETTY IMAGES

BY MONICA CHIN

FEB 2018

The secret to identifying certain health conditions may be hidden in our eyes.

Researchers from Google and its health-tech subsidiary Verily announced on Monday that they have successfully created algorithms to predict whether someone has high blood pressure or is at risk of a heart attack or stroke simply by scanning a person's eyes, the *Washington Post* reports.

SEE ALSO: This fork helps you stay healthy

Google's researchers trained the algorithm with images of scanned retinas from more than 280,000 patients. By reviewing this massive database, Google's algorithm trained itself to recognize the patterns that designated people as at-risk.

This algorithm's success is a sign of exciting developments in healthcare on the horizon. As Google fine-tunes the technology, it could one day

AlphaGo (2015)

First program to beat a professional Go player

AlphaZero (2017)

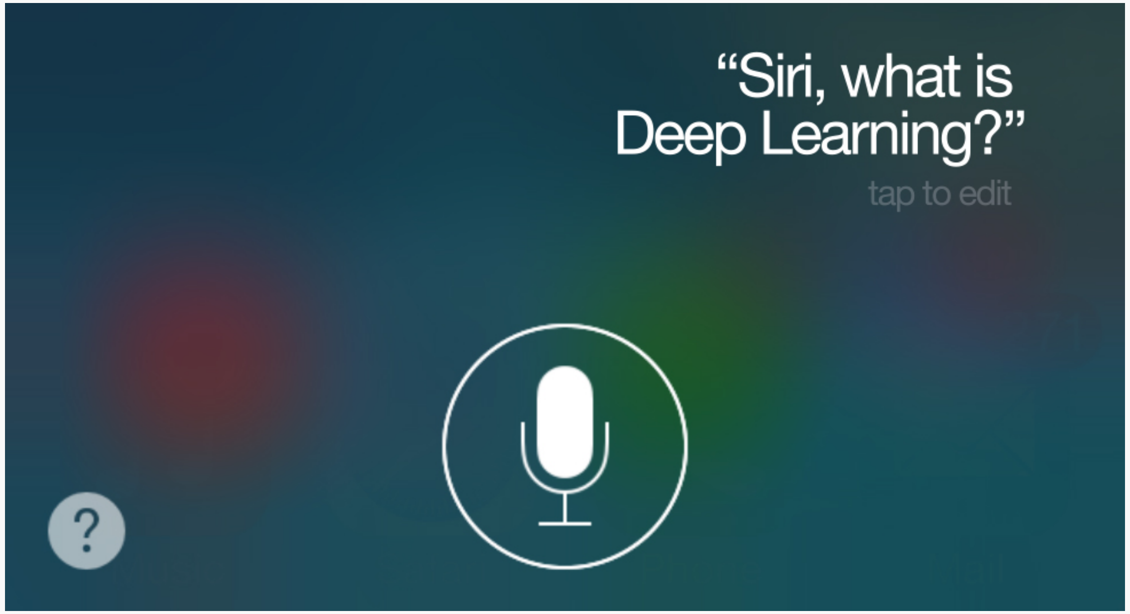
DeepMind

AlphaZero AI beats champion chess program after teaching itself in four hours

Google's artificial intelligence sibling DeepMind repurposes Go-playing AI to conquer chess and shogi without aid of human knowledge

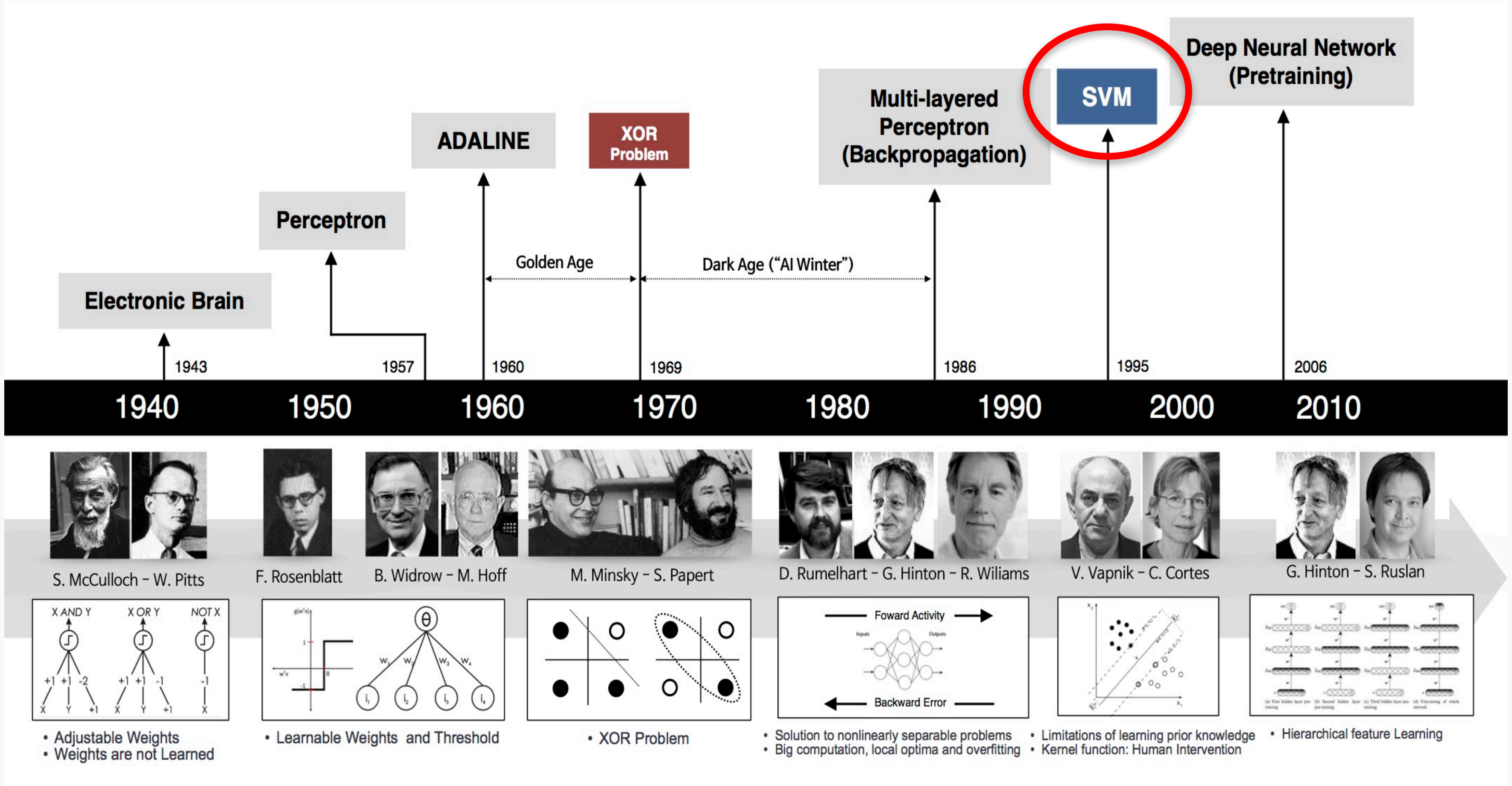
iOS Speech Synthesis (2016-)

Trained from 20 hours of high quality speech



machinelearning.apple.com

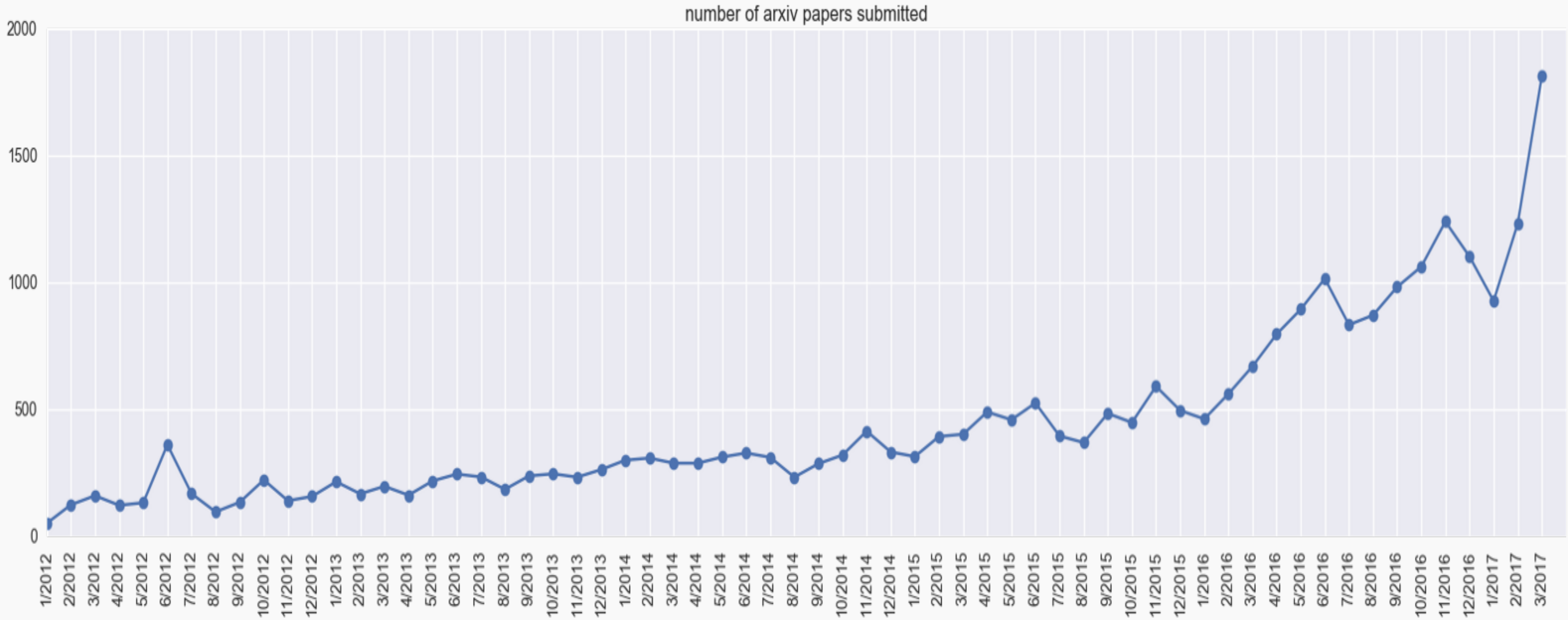
Historical Trends



beamandrew.github.io/

Historical Trends

ArXiv papers on deep learning: 2012-2017



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Classification and Logistic Regression

Regression and Classification

Methods that are centered around modeling and prediction of a **quantitative** response variable (ex, number of taxi pickups, number of bike rentals, etc) are called **regressions** (and Ridge, LASSO, etc).

When the response variable is **categorical**, then the problem is no longer called a regression problem but is instead labeled as a **classification problem**.

The goal is to attempt to classify each observation into a category (aka, class or cluster) defined by Y, based on a set of predictor variables X.

Typical Classification Examples

The motivating examples for this lecture(s), homeworks and labs are based on classification. Classification problems are common in these domains:

- Trying to determine where to set the *cut-off* for some diagnostic test (pregnancy tests, prostate or breast cancer screening tests, etc…)
- Trying to determine if cancer has gone into remission based on treatment and various other indicators
- Trying to classify patients into types or classes of disease based on various genomic markers

Data: Response vs. Predictor Variables

X
predictors
features
covariates

Y
outcome
response variable
dependent variable

TV	radio	newspaper	sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	9.3
151.5	41.3	58.5	18.5
180.8	10.8	58.4	12.9

n observations

p predictors

Response vs. Predictor Variables

$X = X_1, \ldots, X_p$

$X_j = x_{1j}, \ldots, x_{ij}, \ldots, x_{nj}$

predictors

features

covariates

$Y = y_1, \ldots, y_n$

outcome

response variable

dependent variable

n **observations**

TV	radio	newspaper	sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	9.3
151.5	41.3	58.5	18.5
180.8	10.8	58.4	12.9

p **predictors**

Heart Data

response variable ***Y*** is Yes/No

Age	Sex	ChestPain	RestBP	Chol	Fbs	RestECG	MaxHR	ExAng	Oldpeak	Slope	Ca	Thal	AHD
63	1	typical	145	233	1	2	150	0	2.3	3	0.0	fixed	No
67	1	asymptomatic	160	286	0	2	108	1	1.5	2	3.0	normal	Yes
67	1	asymptomatic	120	229	0	2	129	1	2.6	2	2.0	reversable	Yes
37	1	nonanginal	130	250	0	0	187	0	3.5	3	0.0	normal	No
41	0	nontypical	130	204	0	2	172	0	1.4	1	0.0	normal	No

Heart Data

These data contain a binary outcome HD for 303 patients who presented with chest pain. An outcome value of:

- ***Yes*** indicates the presence of heart disease based on an angiographic test,
- ***No*** means no heart disease.

There are 13 predictors including:

- Age
- Sex
- Chol (a cholesterol measurement),
- MaxHR
- RestBP

and other heart and lung function measurements.

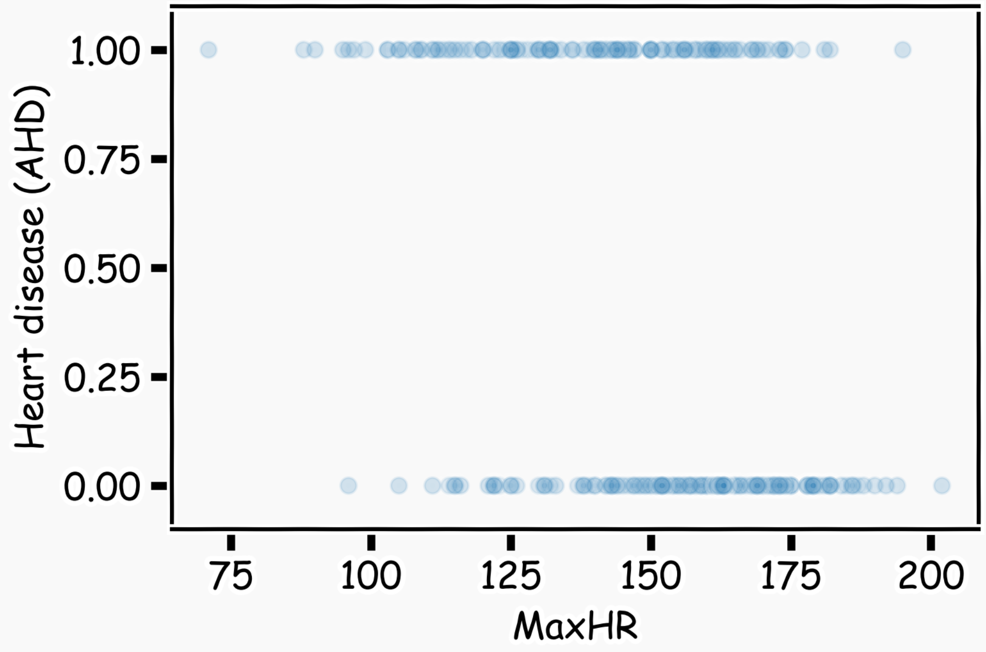
Logistic Regression

Logistic Regression addresses the problem of estimating a probability, $P(y = 1)$, given an input X. The logistic regression model uses a function, called the ***logistic*** function, to model $P(y = 1)$:

$$P(Y = 1) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}} = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X)}}$$

Heart Data: logistic estimation

We'd like to predict whether or not a person has a heart disease. And we'd like to make this prediction, for now, just based on the MaxHR.



Logistic Regression

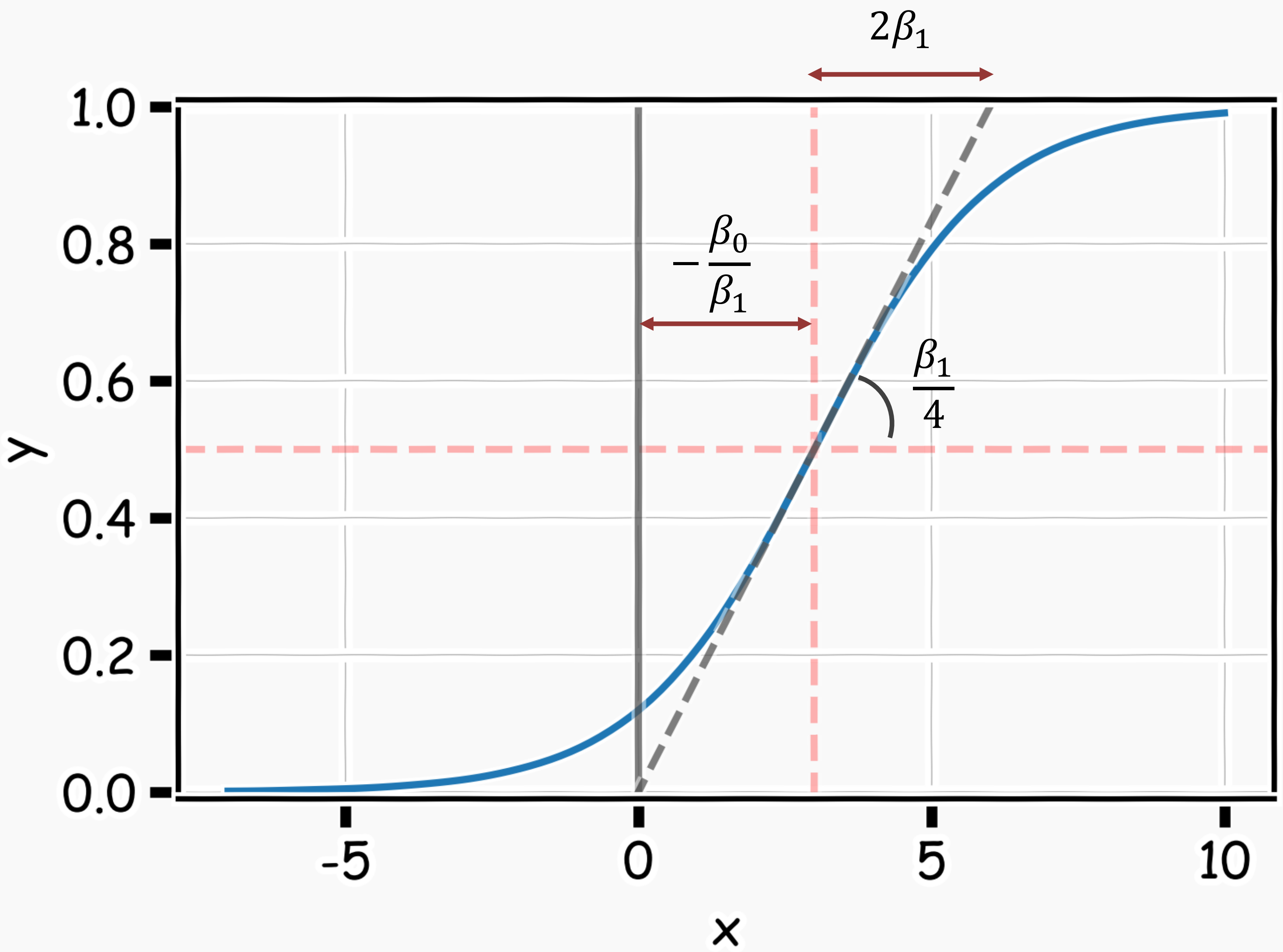
As a result the model will predict $P(y=1)$ with an S-shaped curve, which is the general shape of the logistic function.

β_0 shifts the curve right or left by $c = -\frac{\beta_0}{\beta_1}$.

β_1 controls how steep the S-shaped curve is distance from ½ to ~1 or ½ to ~0 to ½ is $\frac{2}{\beta_1}$

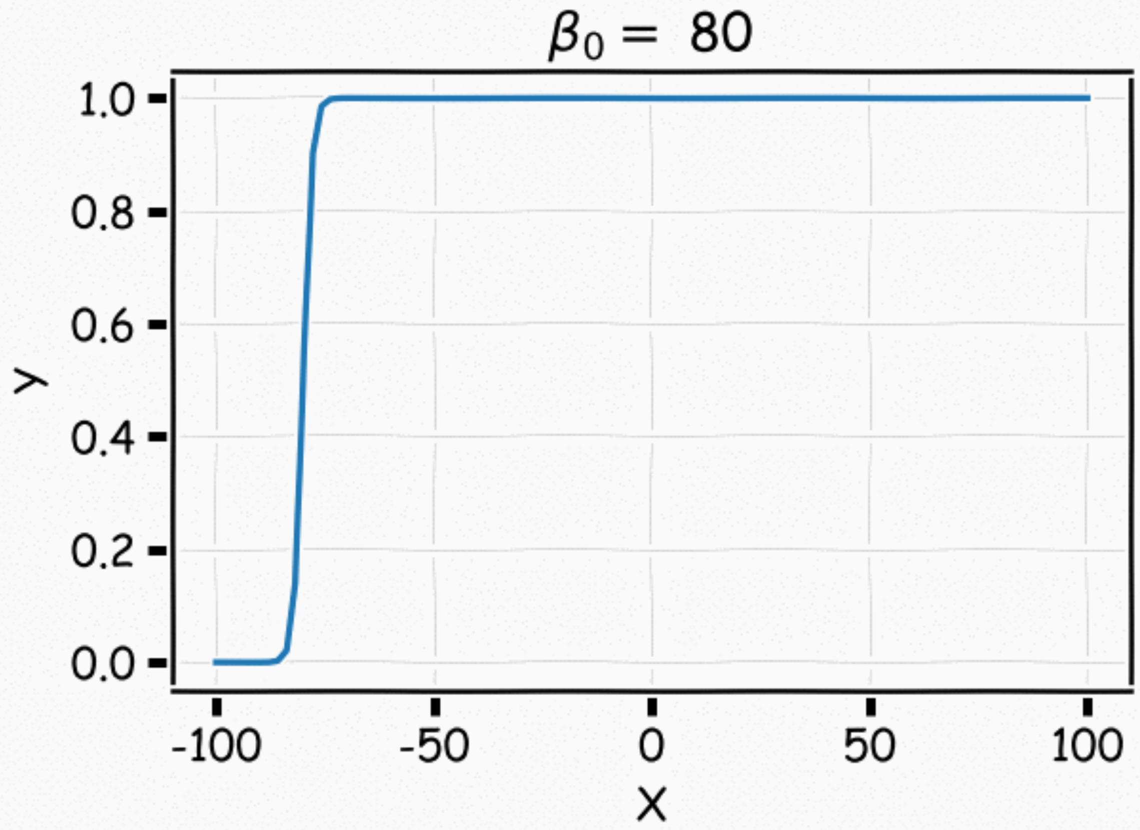
Note: if β_1 is positive, then the predicted $P(y=1)$ goes from zero for small values of X to one for large values of X and if β_1 is negative, then has the $P(y=1)$ opposite association.

Logistic Regression



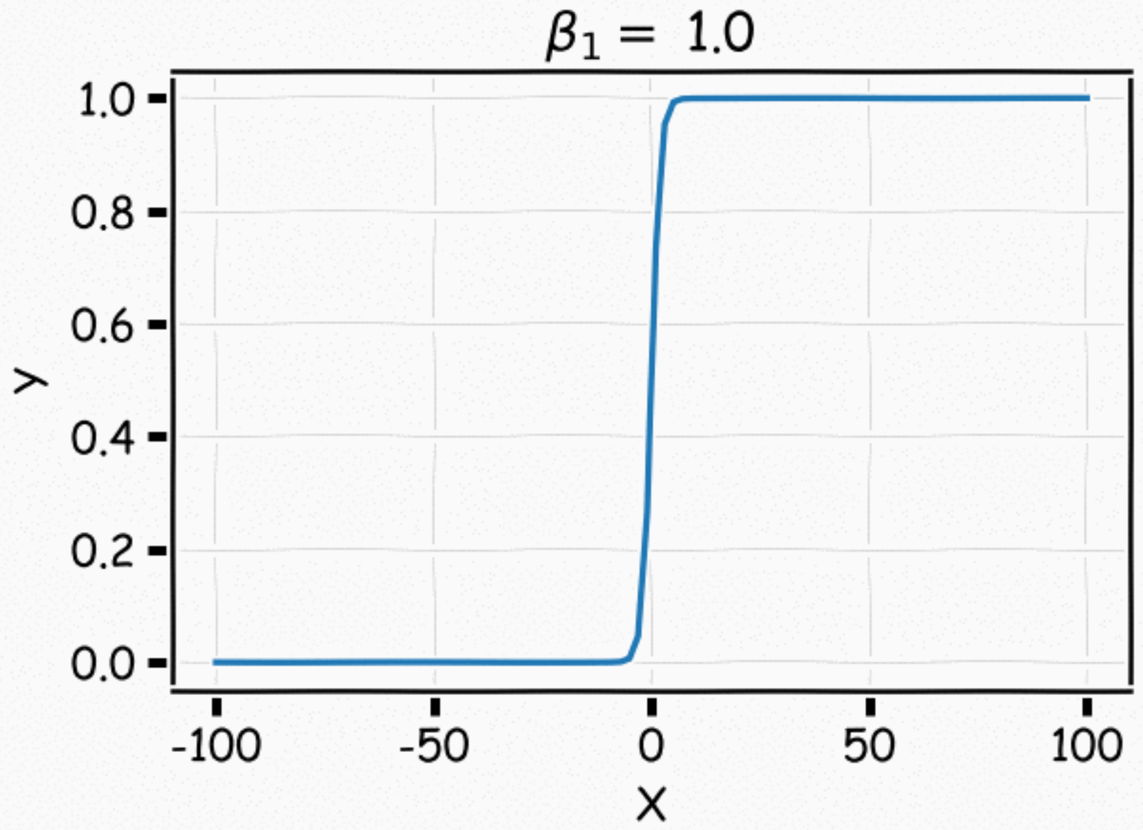
Logistic Regression

$$P(Y=1) = \frac{1}{1+e^{-(\beta_0+\beta_1 X)}}$$



Logistic Regression

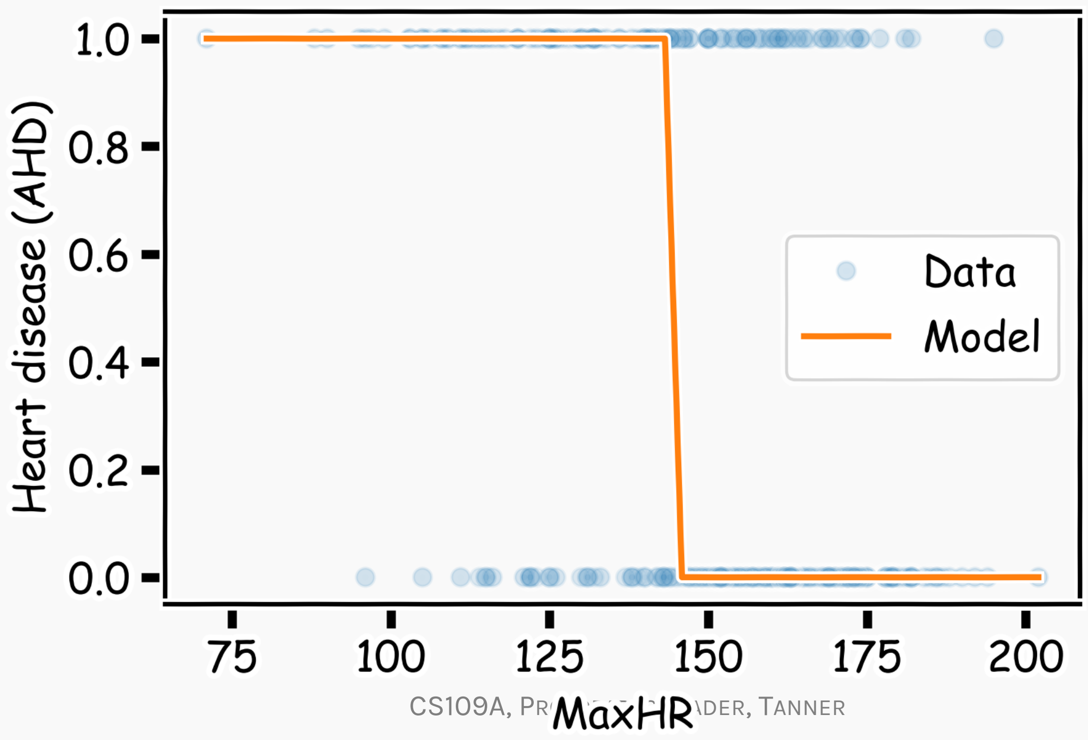
$$P(Y=1) = \frac{1}{1+e^{-(\beta_0+\beta_1 X)}}$$



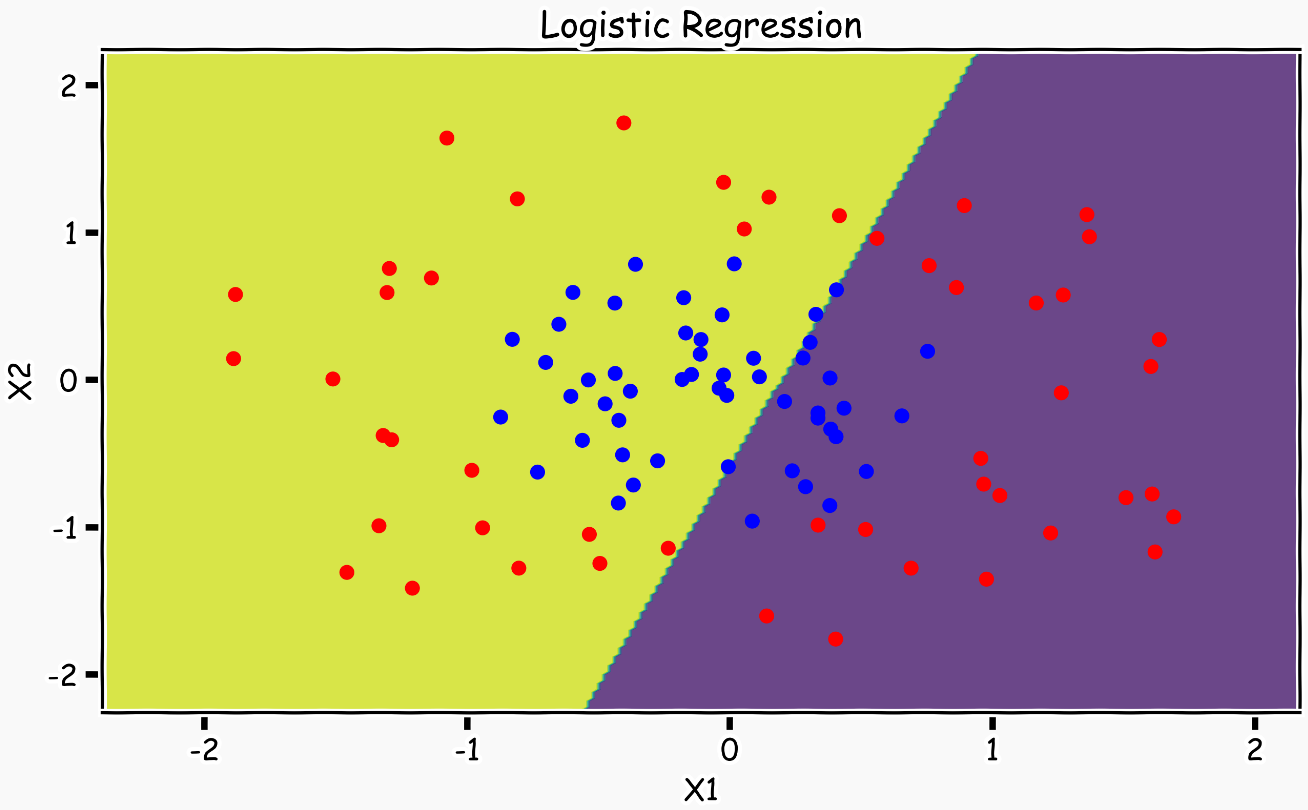
Estimating the coefficients for Logistic Regression

Find the coefficients that minimize the loss function

$$\mathcal{L}(\beta_0, \beta_1) = -\sum_i [y_i \log p_i + (1 - y_i)\log(1 - p_i)]$$

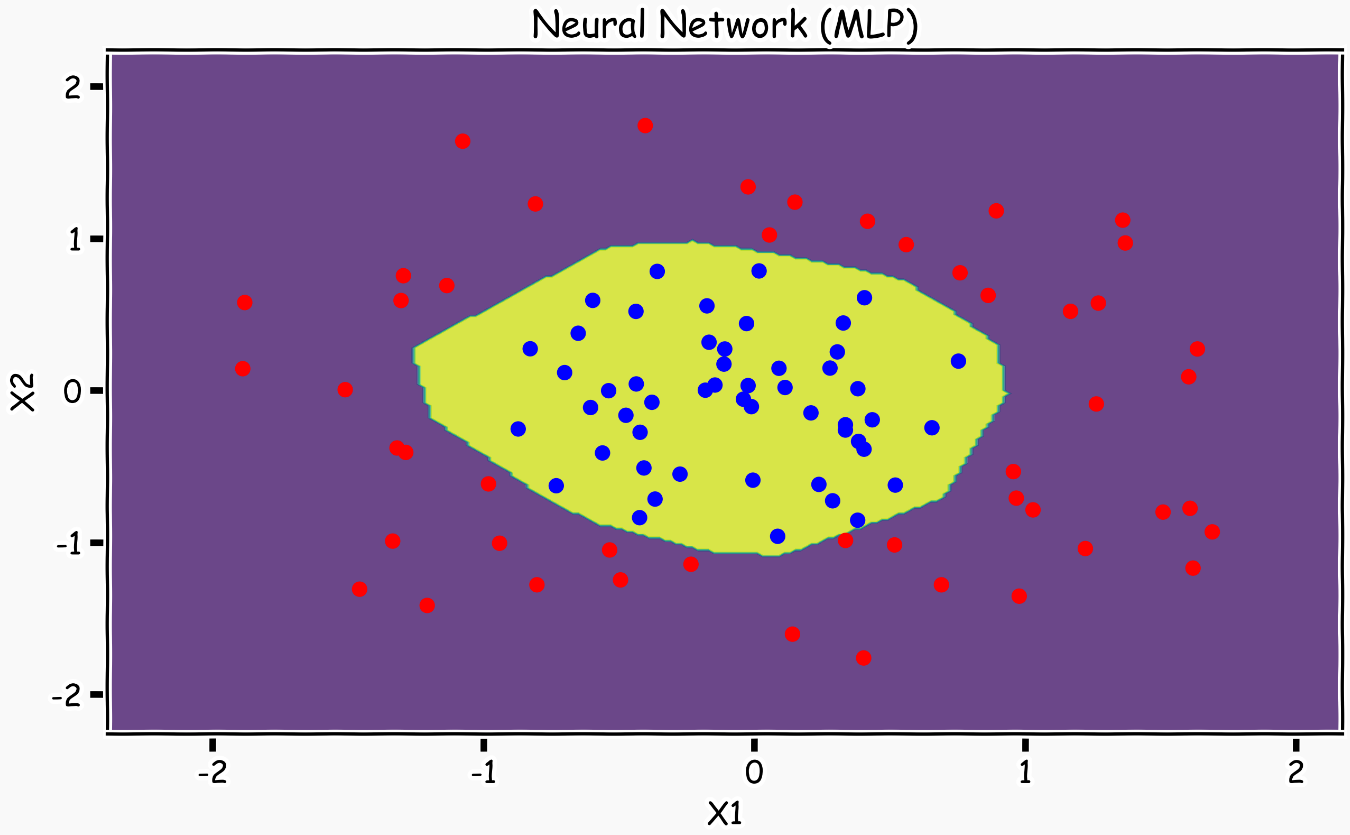


Need for Non-Linearity



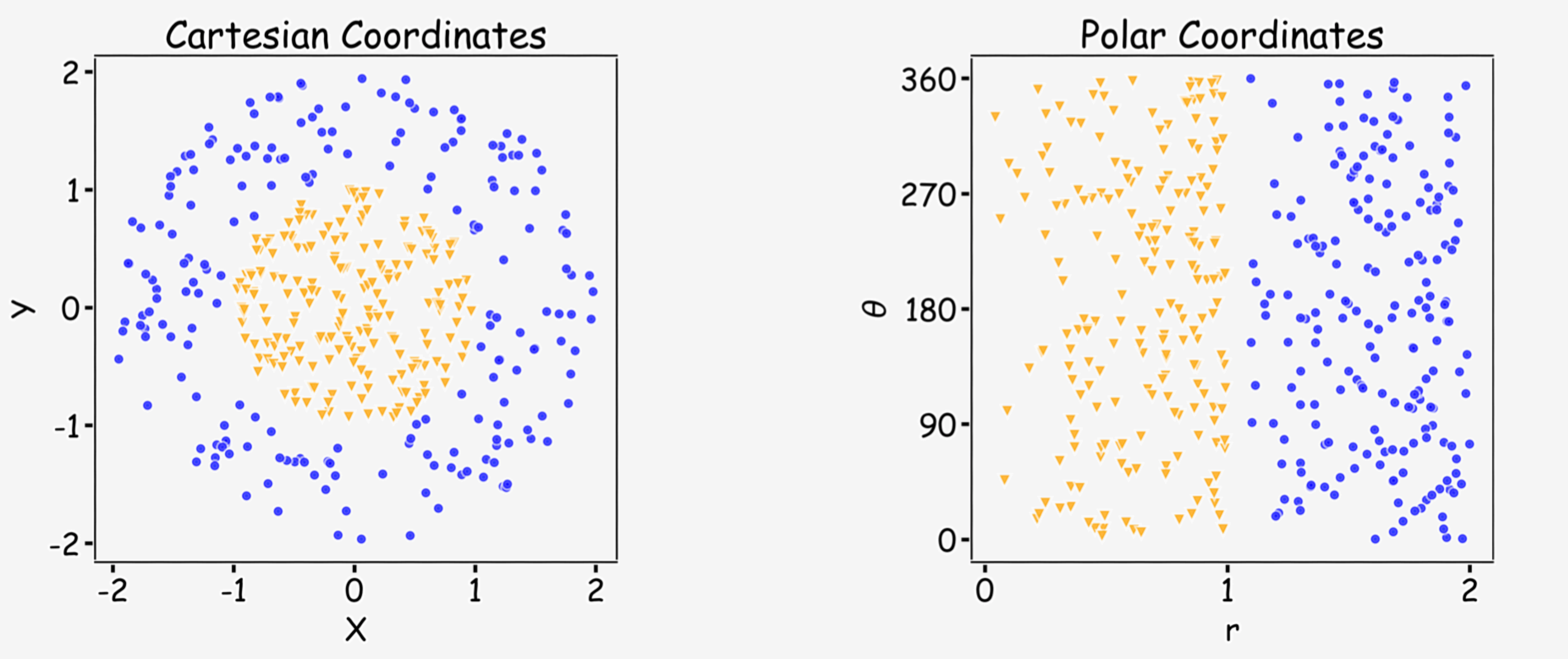
Without **augmenting the features** (i.e. without adding $X1^2$ or $X2^2$ non-linear features), **Logistic Regression is incapable of modeling the correct decision boundary.**

Neural Networks to The Rescue



A **neural network is a powerful non-linear** model that can easily **model the non-linear decision boundary correctly.**

Representation Matters



Neural networks can **learn useful representations** for the problem. This is another reason why they can be so powerful!

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Logistic Regression Revisited

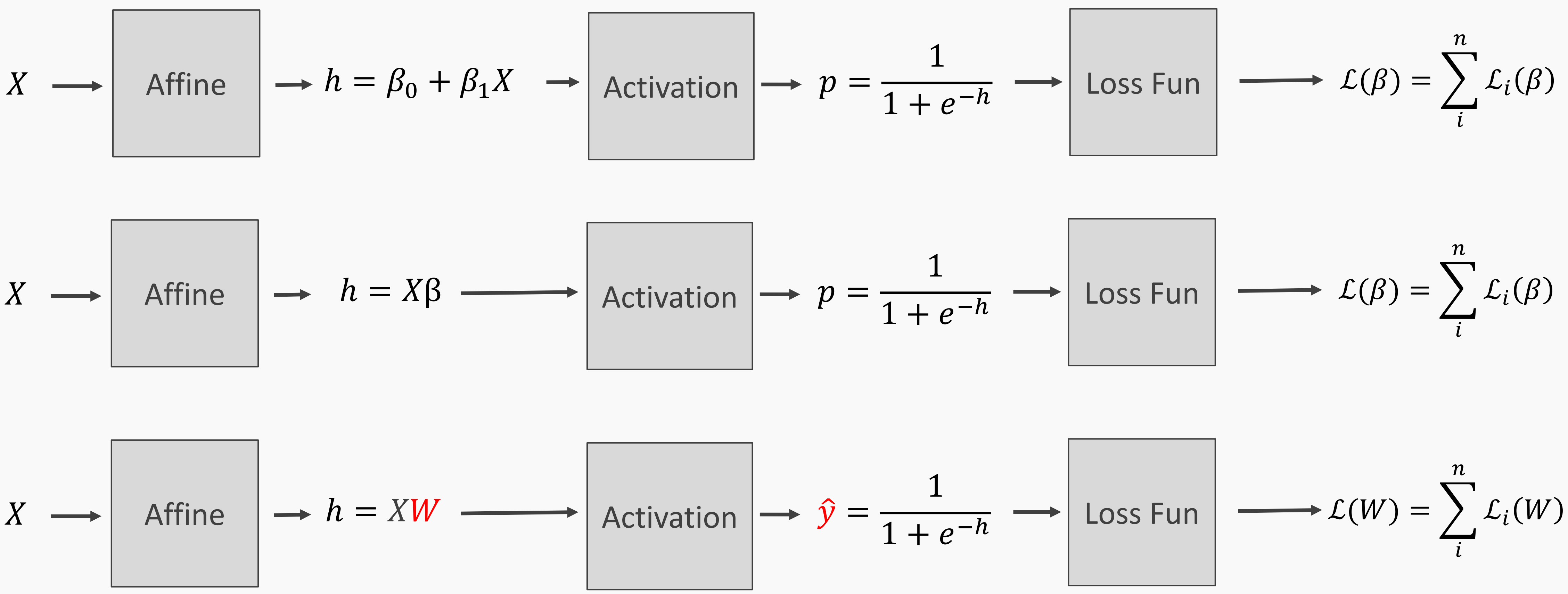
x_i → Affine → $h_i = \beta_0 + \beta_1 x_i$ → Activation → $p_i = \frac{1}{1+e^{-h_i}}$ → Loss Fun → $\mathcal{L}_i(\beta) = -y_i \ln(p_i) - (1-y_i)\ln(1-p_i)$

x_k → Affine → $h_k = \beta_0 + \beta_1 x_k$ → Activation → $p_i = \frac{1}{1+e^{-h_k}}$ → Loss Fun → $\mathcal{L}_k(\beta) = -y_k \ln(p_k) - (1-y_k)\ln(1-p_k)$

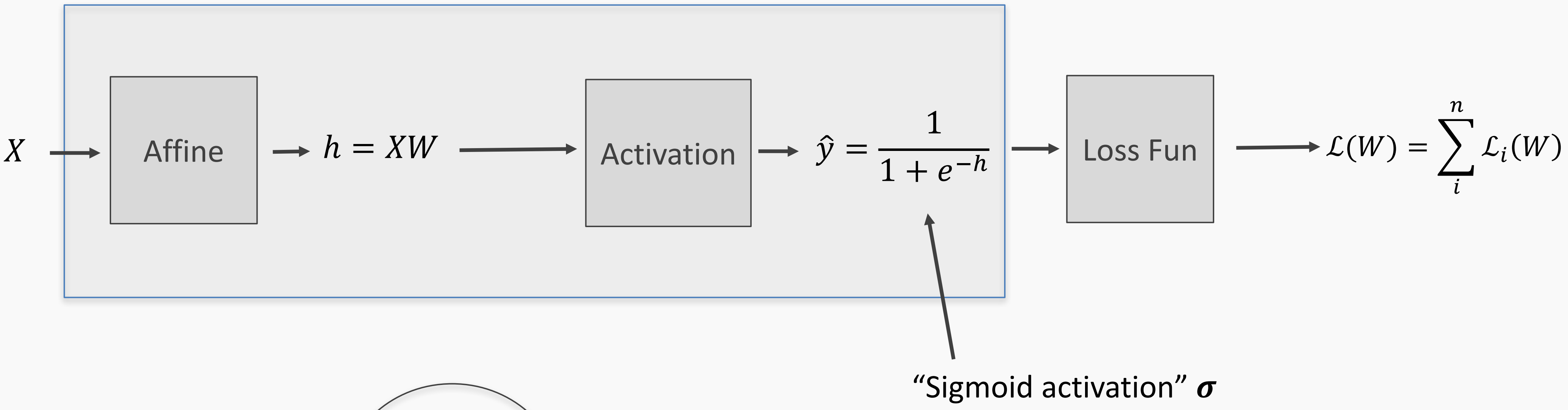
⋮

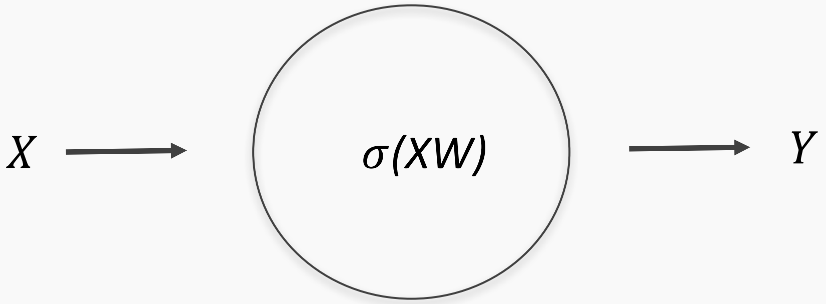
X → Affine → $h = \beta_0 + \beta_1 X$ → Activation → $p = \frac{1}{1+e^{-h}}$ → Loss Fun → $\mathcal{L}(\beta) = \sum_i^n \mathcal{L}_i(\beta)$

Build our first ANN



Build our first ANN





Single Neuron Network

Very similar to Perceptron

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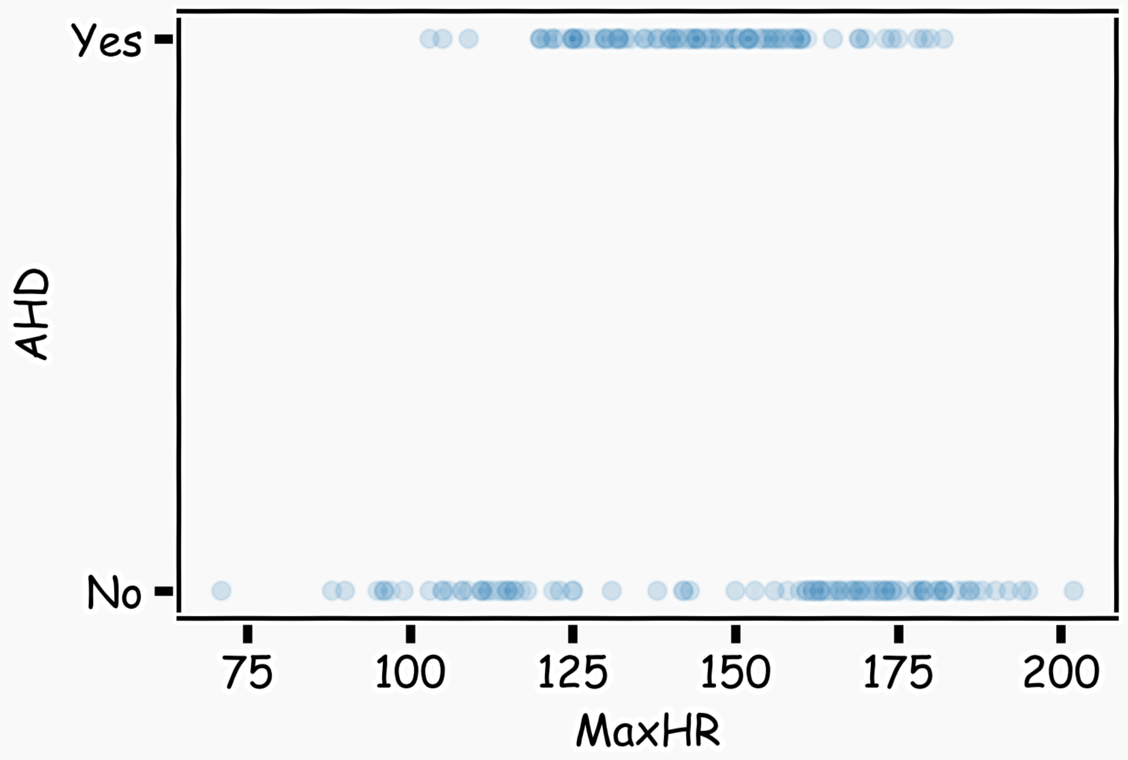
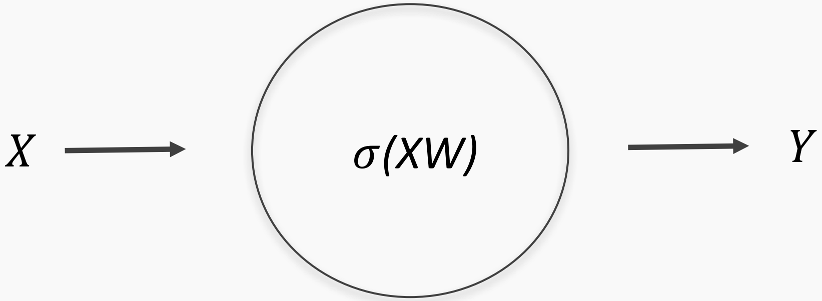
Example Using Heart Data

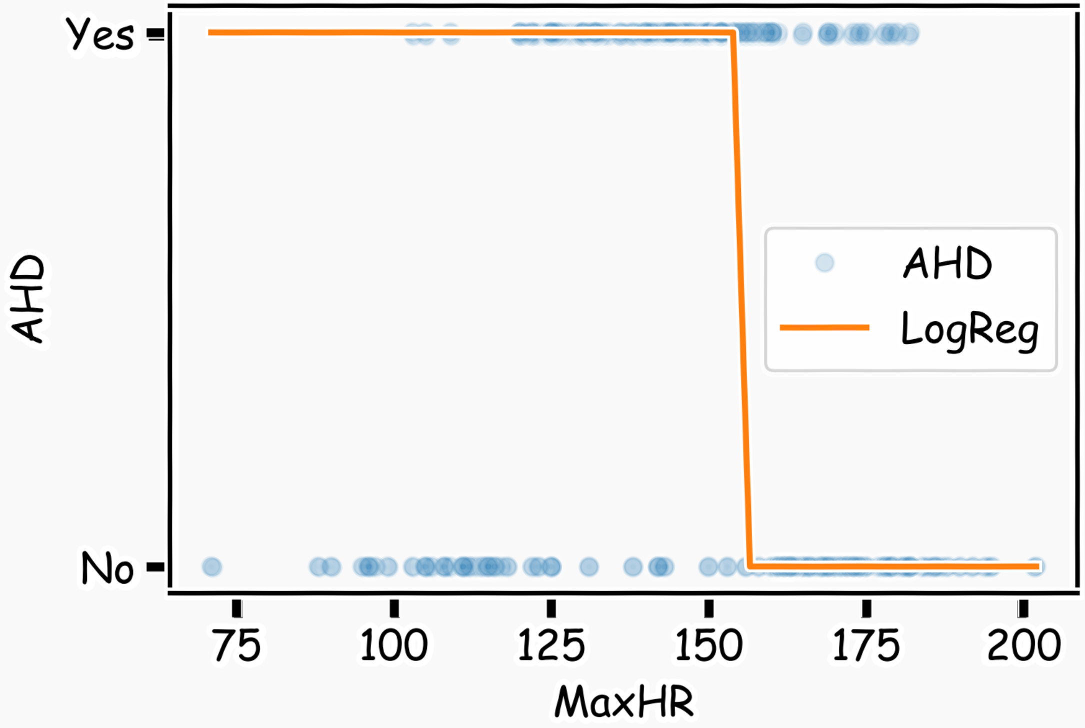
Slightly modified data to illustrate concepts.



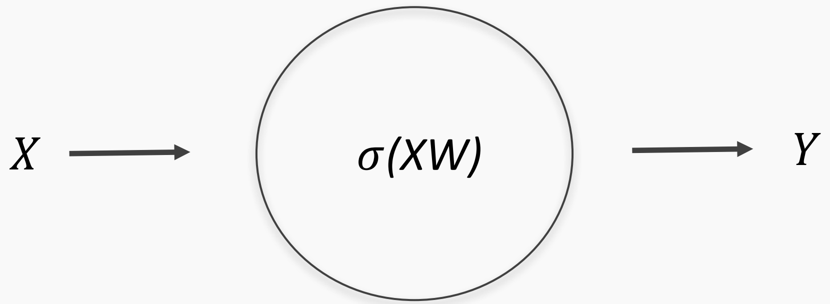
Example Using Heart Data



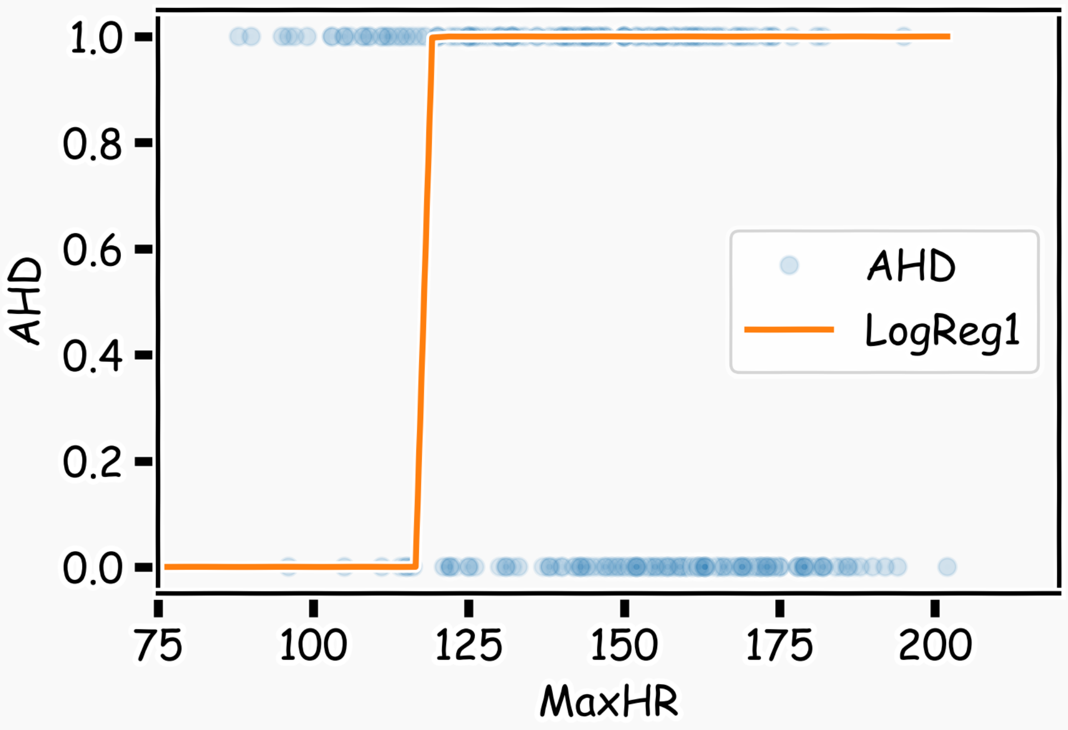
Choose W such as



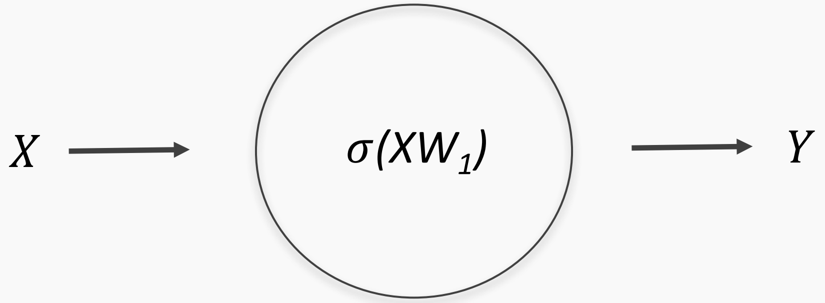
Example Using Heart Data

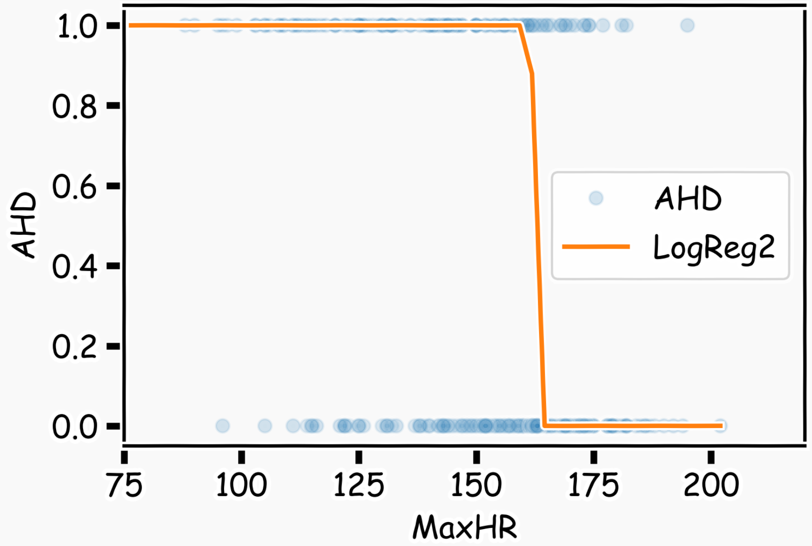


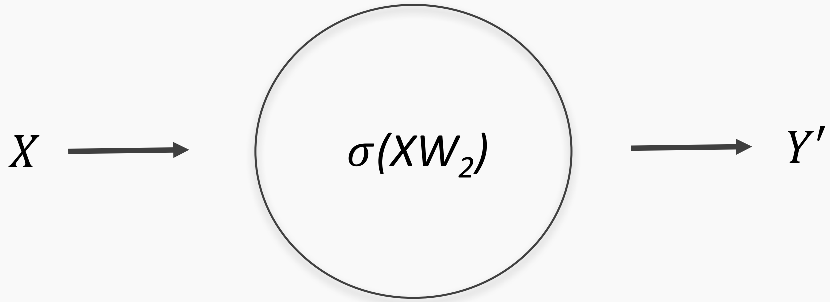
Choose W such as

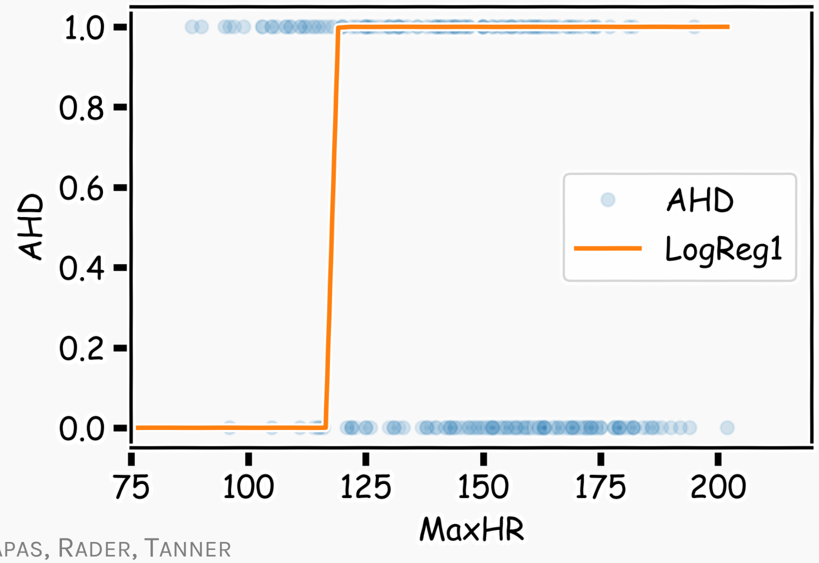


Example

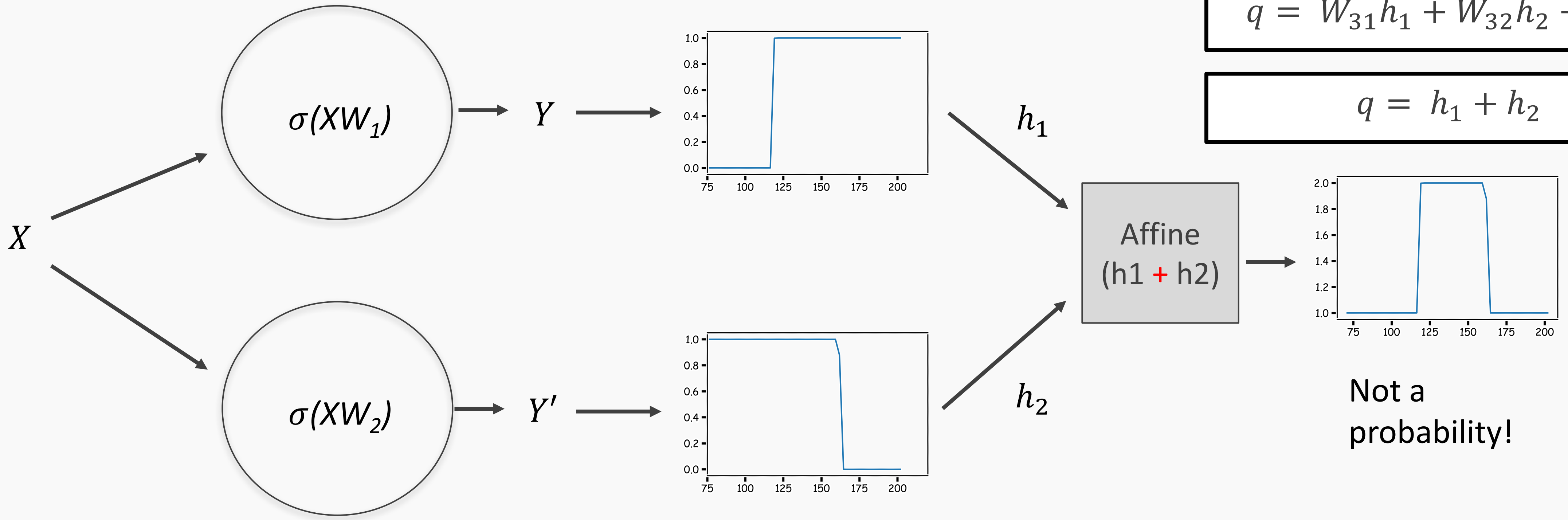

X
σ(XW1)
Y


1.0
0.8
0.6
0.4
0.2
0.0
AHD
75
100
125
150
175
200
MaxHR
AHD
LogReg2


X
σ(XW2)
Y′


1.0
0.8
0.6
0.4
0.2
0.0
AHD
75
100
125
150
175
200
MaxHR
AHD
LogReg1

Pavlos game #232



$$q = W_{31}h_1 + W_{32}h_2 + W_{30}$$

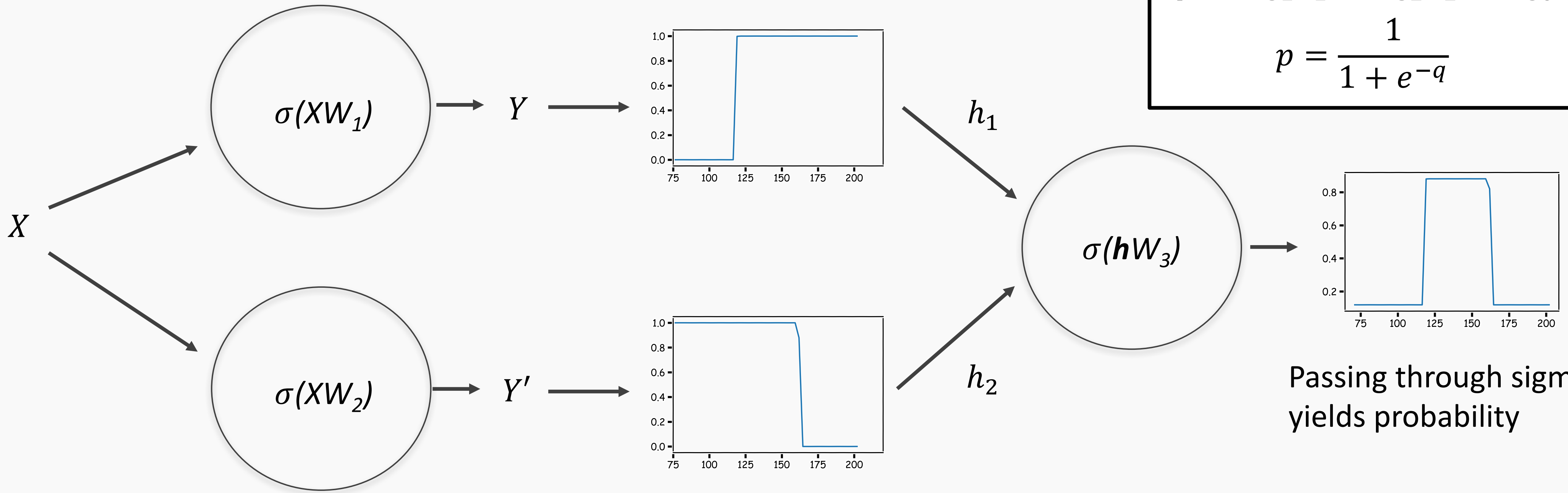
$$q = h_1 + h_2$$

Not a probability!

Pavlos game #232

$$q = W_{31}h_1 + W_{32}h_2 + W_{30}$$

$$p = \frac{1}{1 + e^{-q}}$$

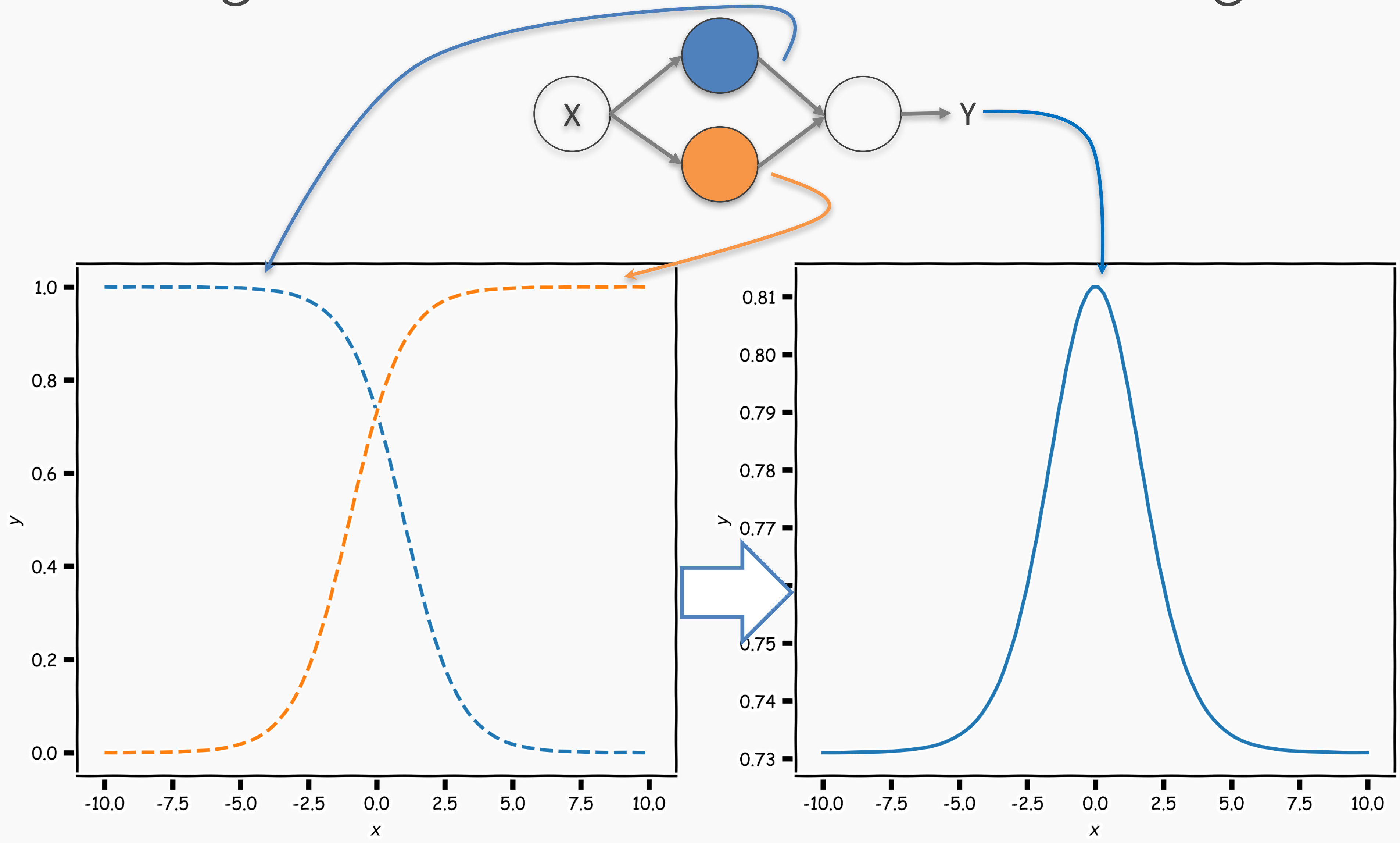


Passing through sigmoid yields probability

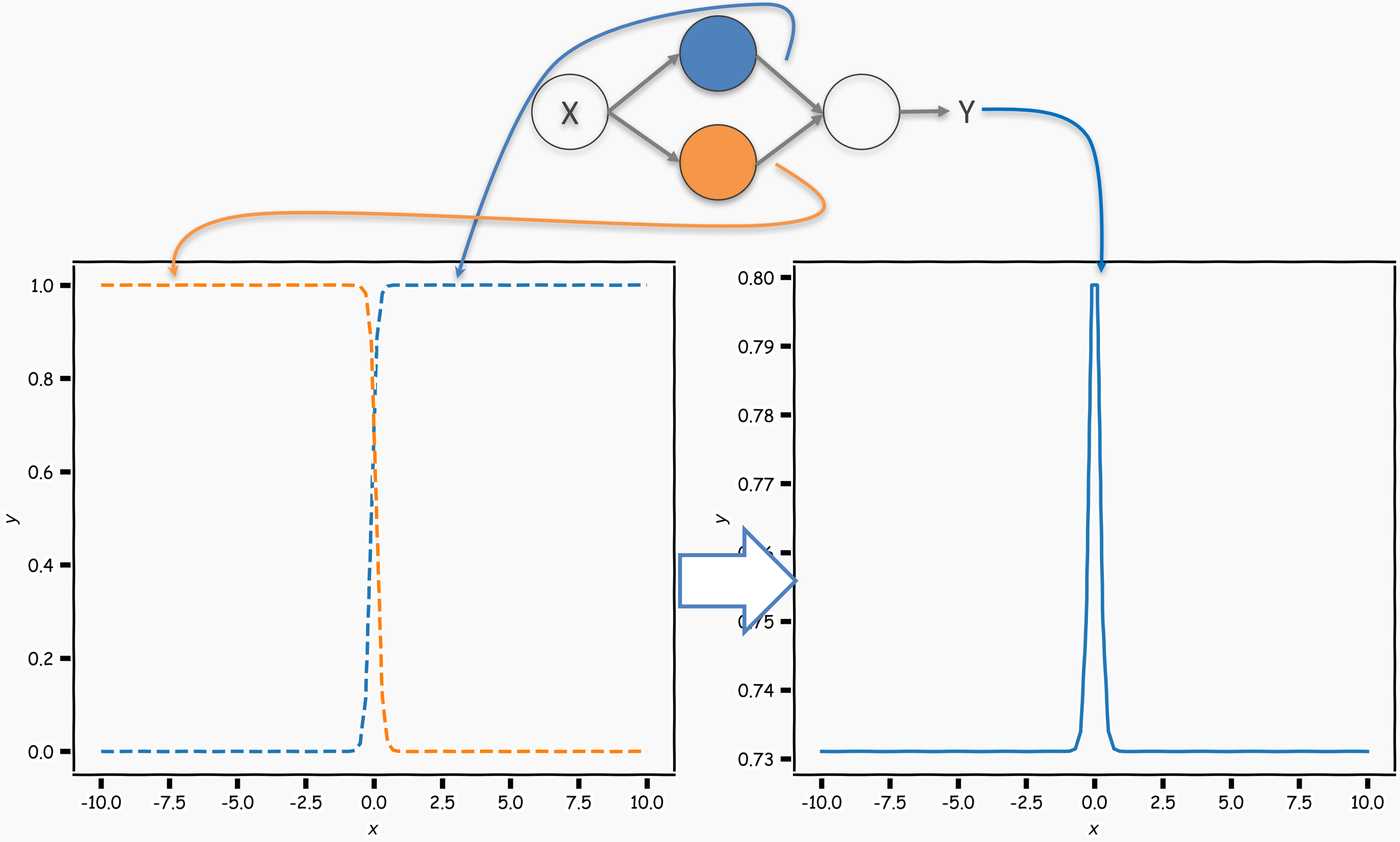
$$L = -y\ \ln(\mathrm{p}) - (1-\mathrm{y})\ln(1-\mathrm{p})$$

Need to learn W1, W2 and W3.

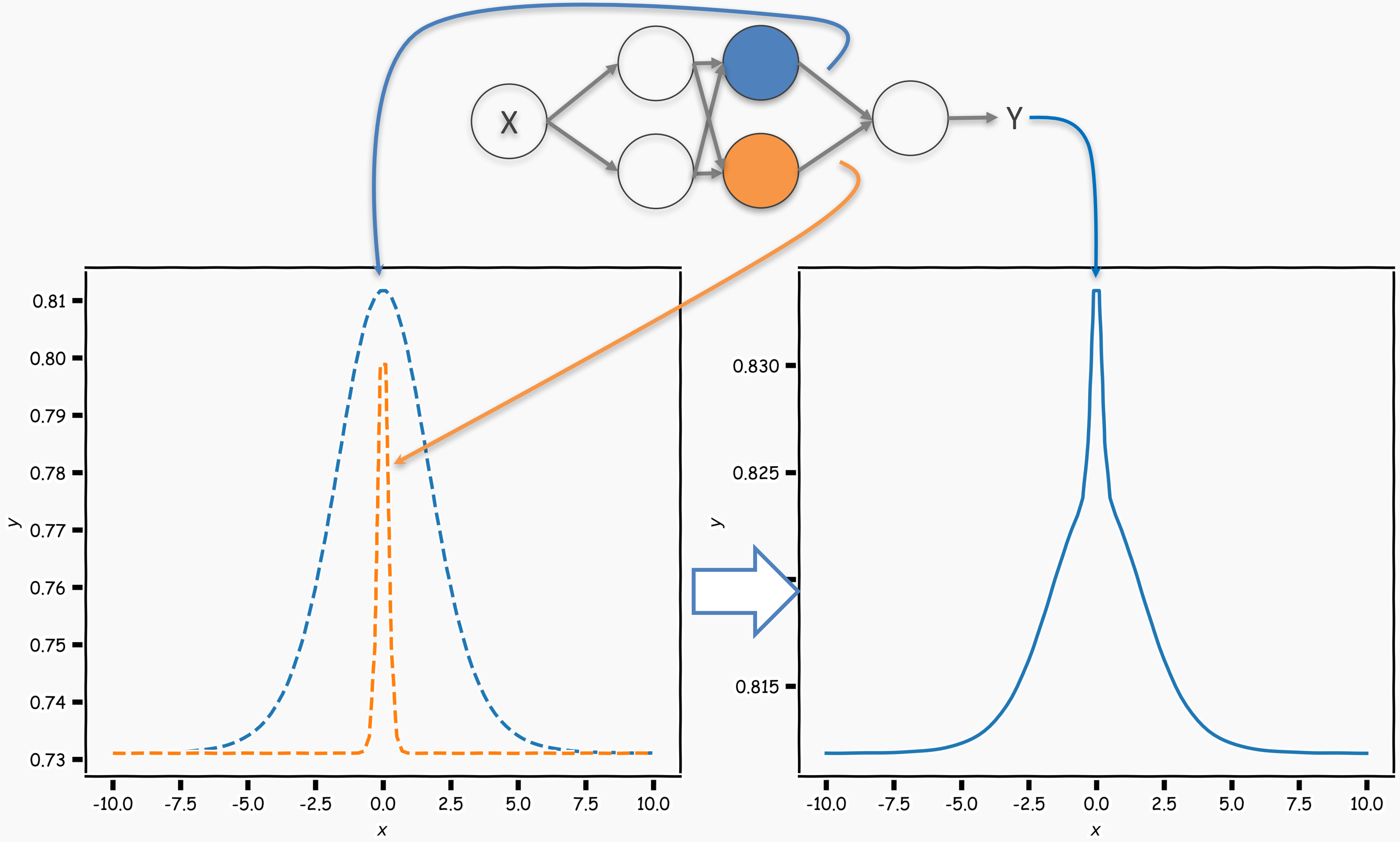
Combining neurons allows us to model interesting functions



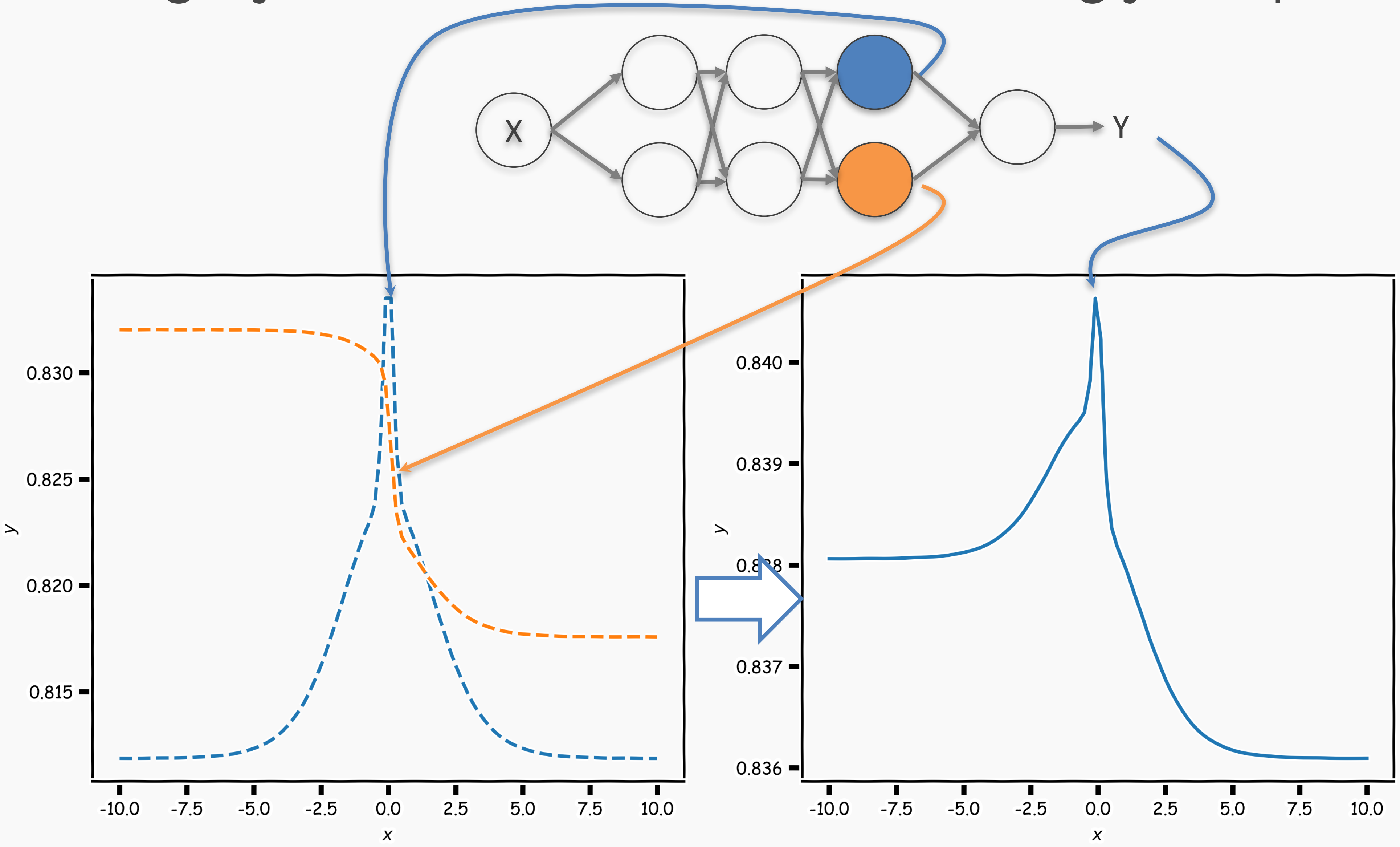
Different weights change the shape and position



Neural networks can model *any* reasonable function



Adding layers allows us to model increasingly complex functions



Summary

So far:

- A single neuron can be a logistic regression unit. We will soon see other choices.
- A neural network is a combination of logistic regression (or other types) units.
- A neural network can approximate non-linear functions.

Next Lecture:

- What kind of activations, how many neurons, how many layers, output unit and loss function?

Following two lecture on NN:

- How do we estimate the weights and biases?
- How to regularize Neural Networks.

Exercises

Quiz (survey)

And two super cool ED exercises