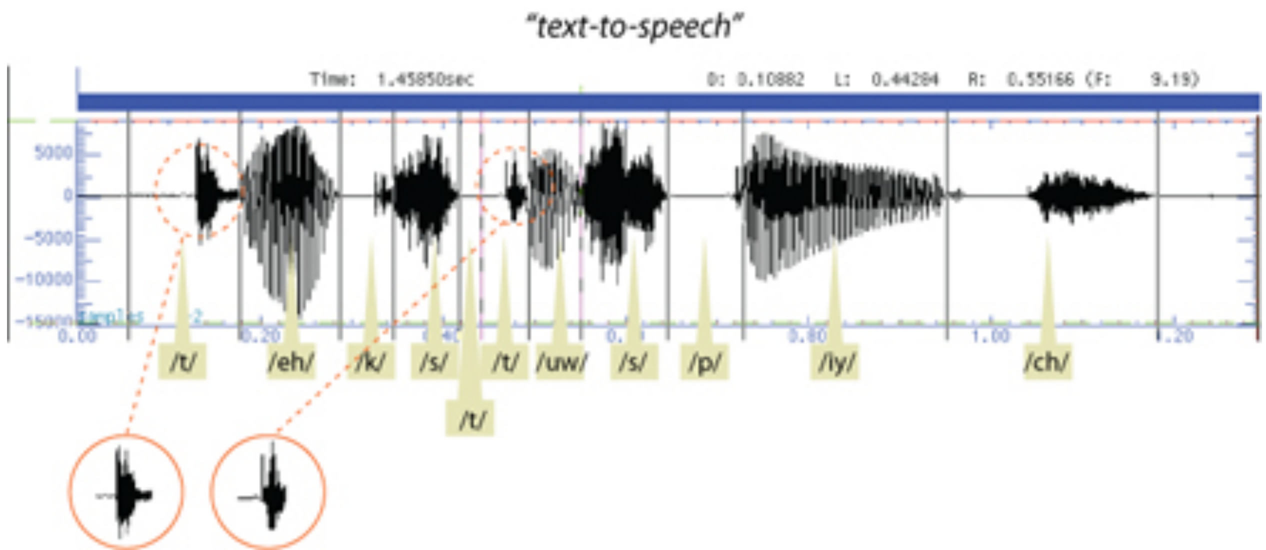
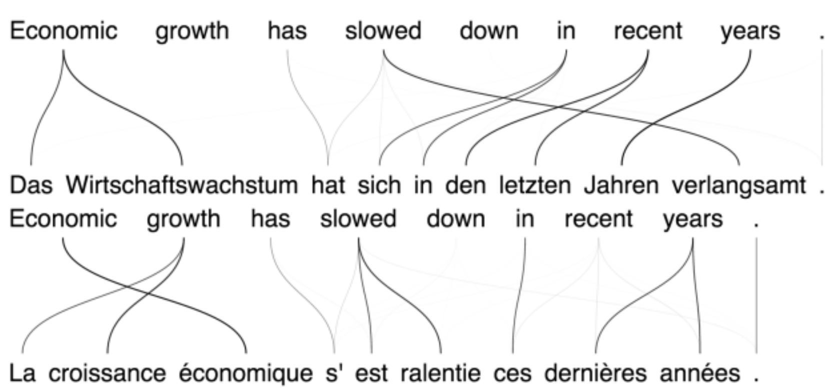
# Lecture 17: RNN

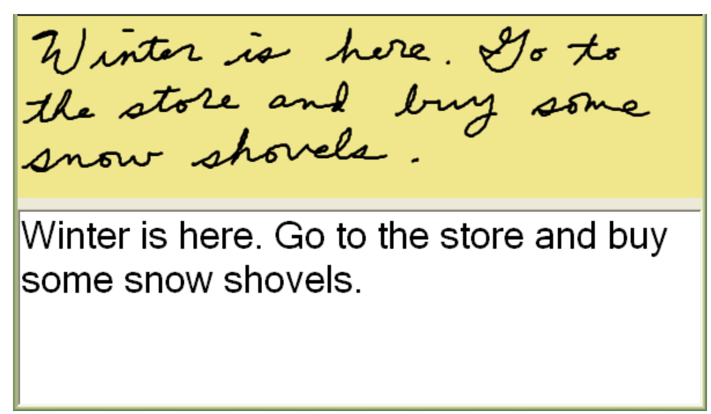
CS 109B, STAT 121B, AC 209B, CSE 109B

Mark Glickman and Pavlos Protopapas

# Sequence Modeling

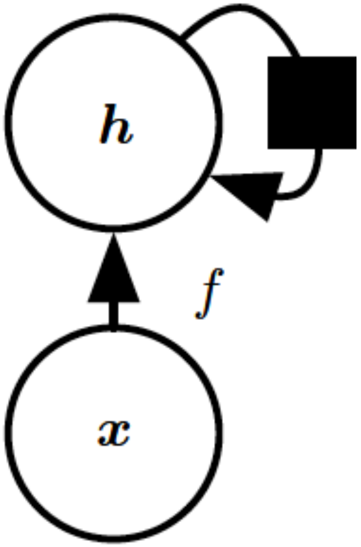


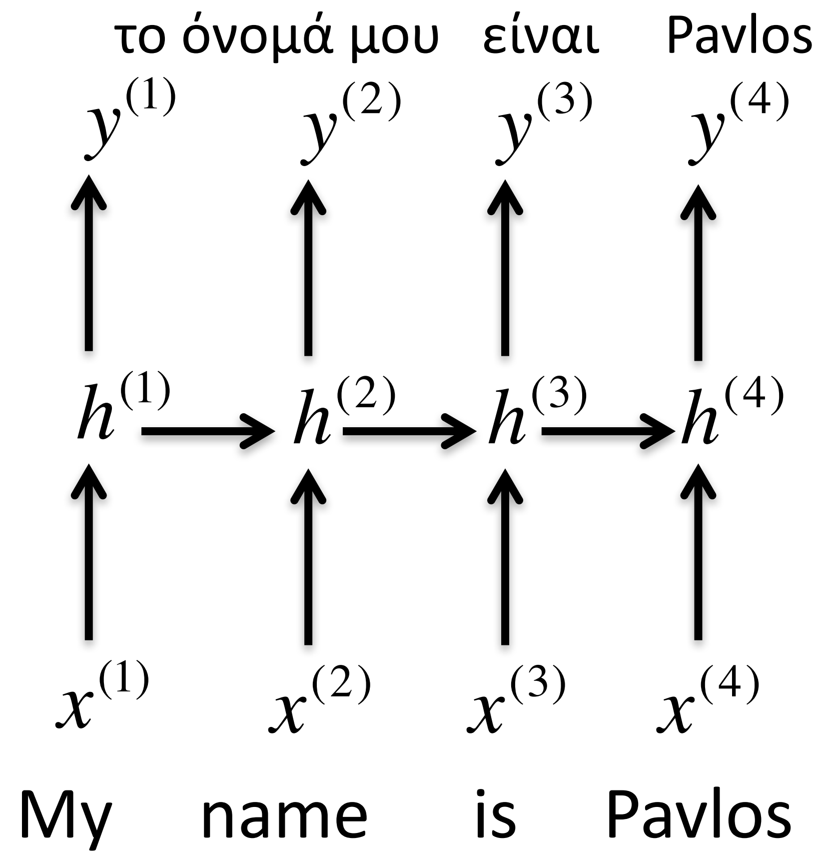



# Recurrent Networks

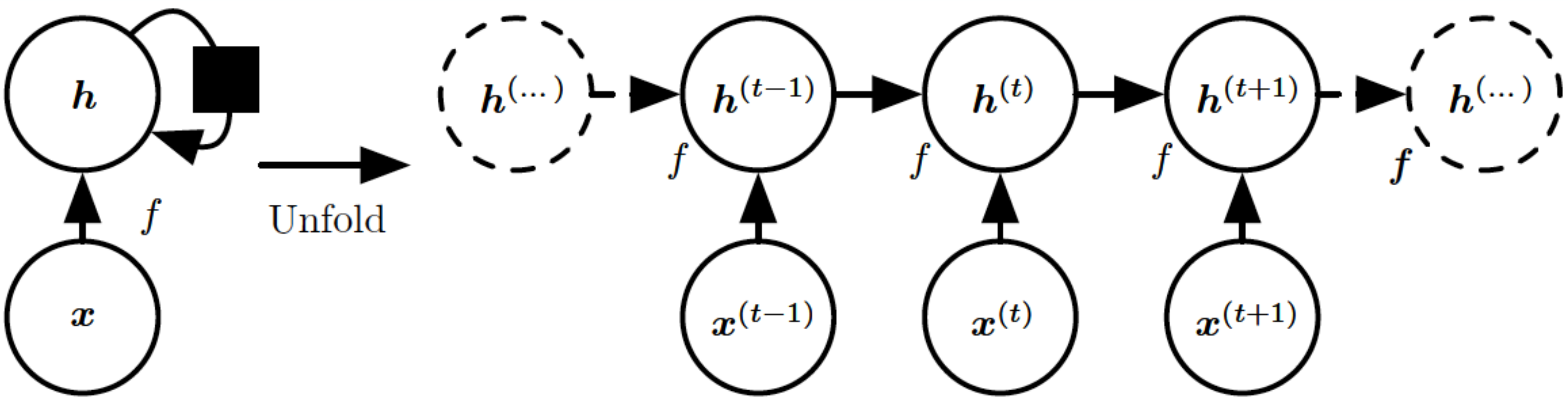
- Image/grid data: convolution networks
- Sequence data: parameter sharing across time



# Example: Machine Translation



# Unfolding the network

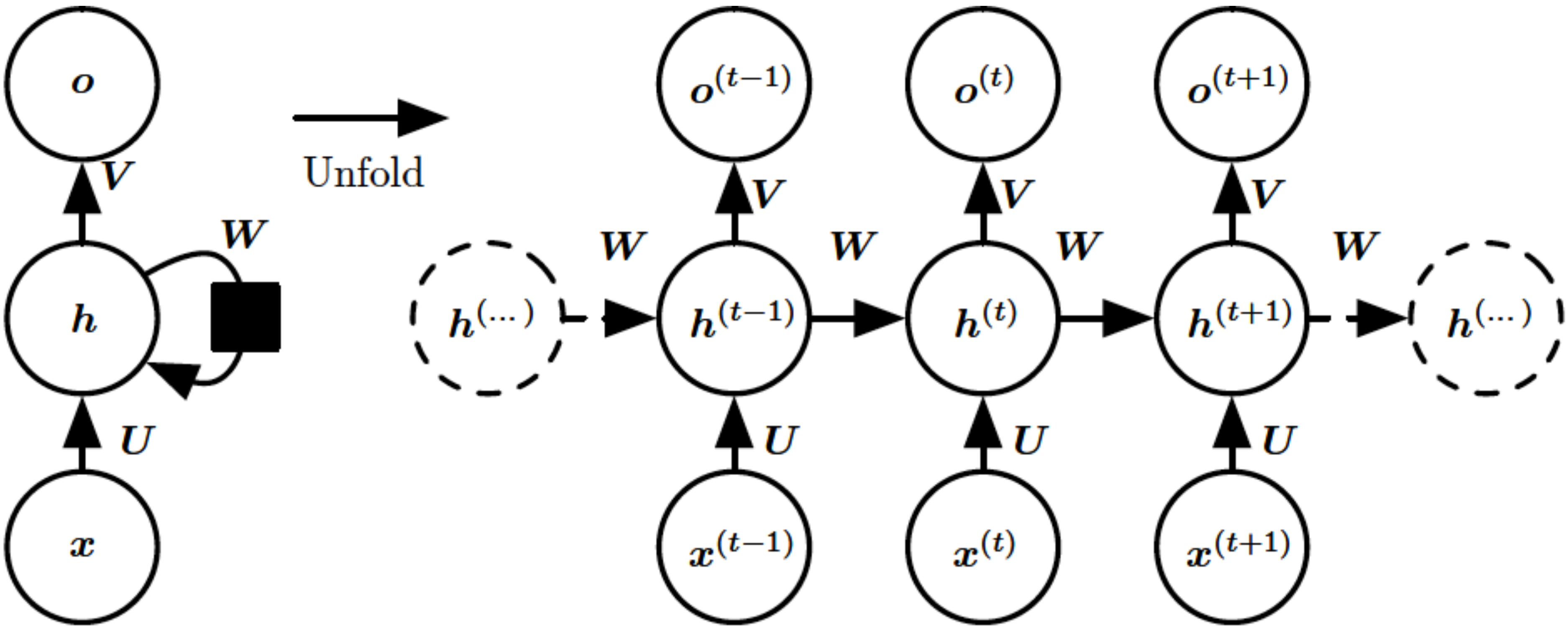



Input sequence

Sequence length may vary for each input

# Hidden-to-hidden Recurrence

E.g. language traslation




Recurrent connections between hidden units

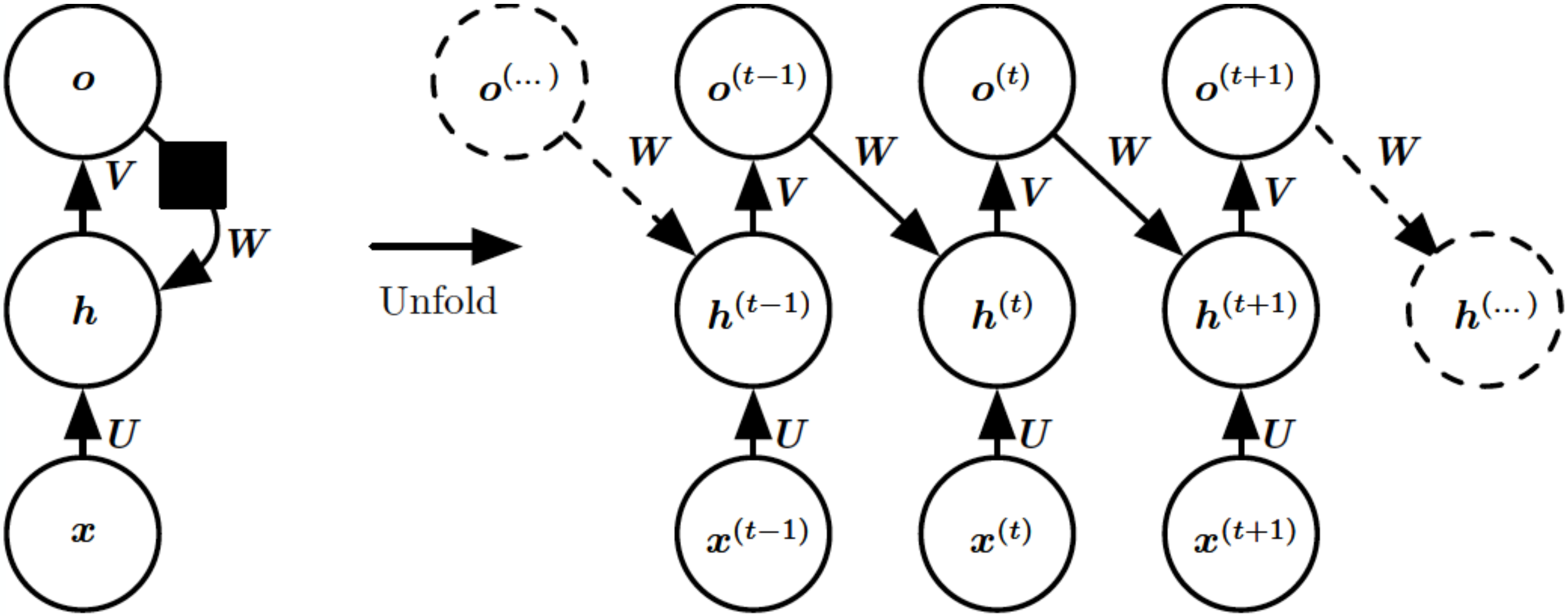
# Hidden-to-hidden Recurrence

$$h^{(t)} = \sigma(\mathbf{W}h^{(t-1)} + \mathbf{U}x^{(t-1)} + b)$$

$$\hat{y}^{(t)} = \mathrm{softmax}(\mathbf{V}h^{(t)} + c)$$

# Output-to-output Recurrence

E.g. auto text completion

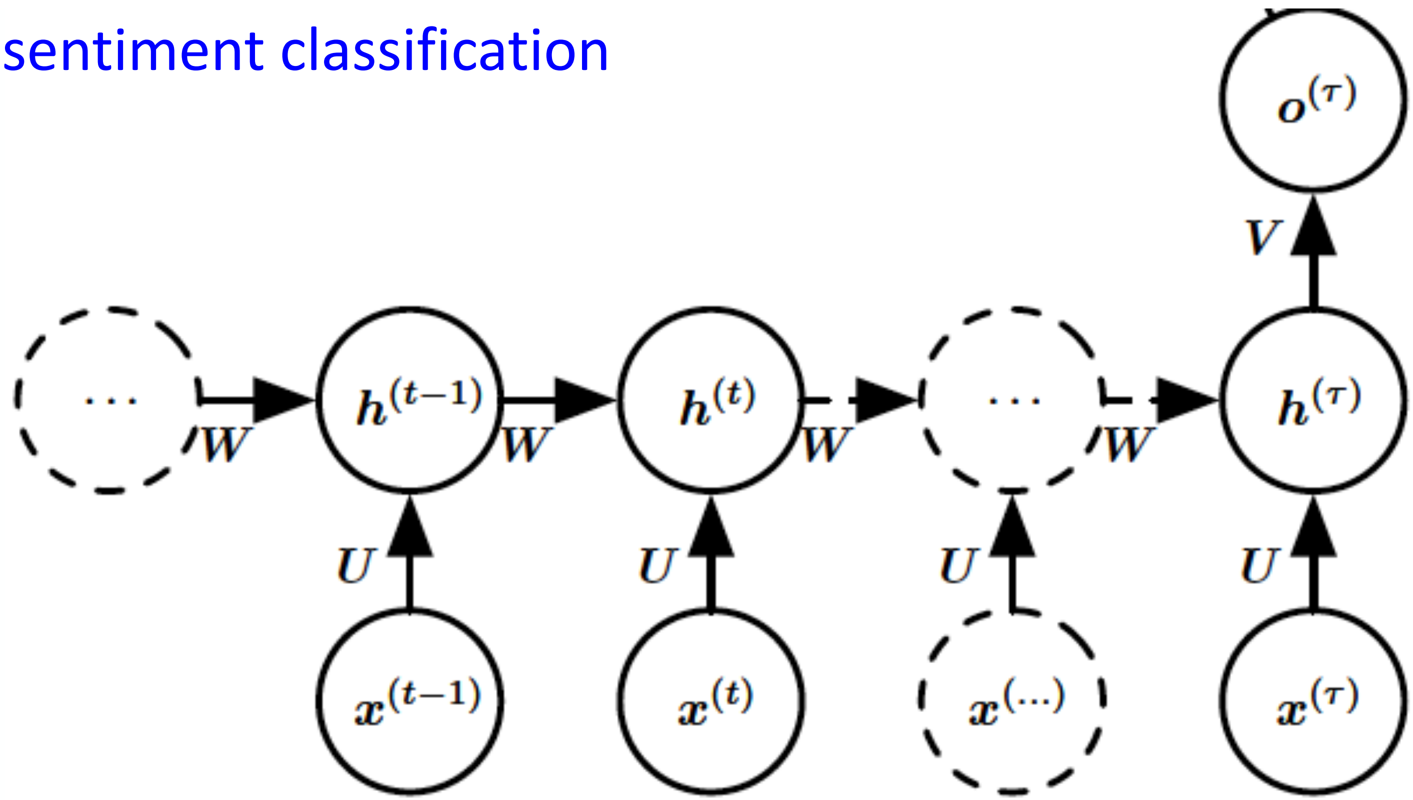



Recurrent connections between output and hidden units

# Single Output RNN

Positive /Negative

E.g. sentiment classification




Product review

Output summarizes input sequence

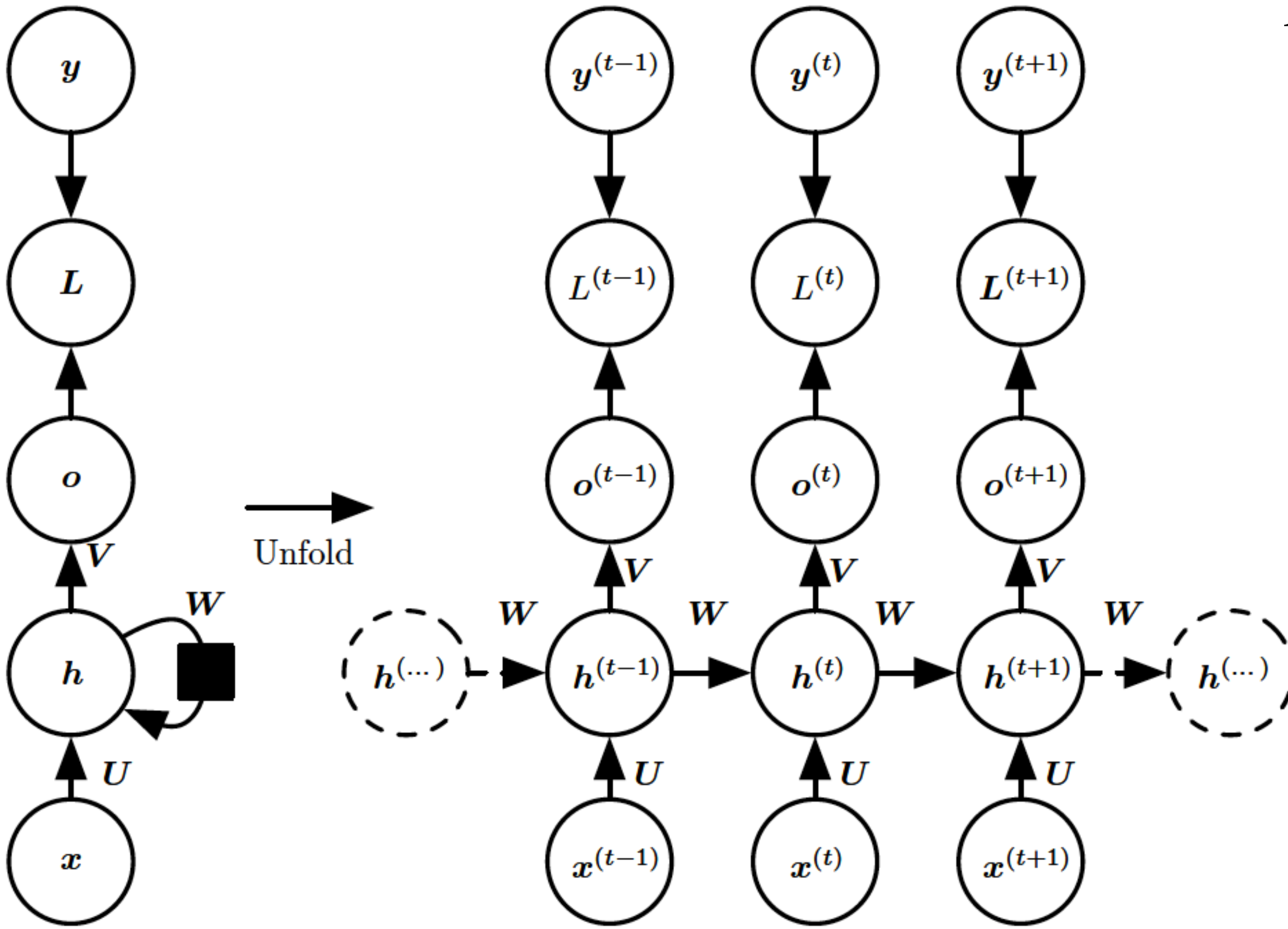
# Outline

- RNN as a graphical model
- RNN training
- Long-term dependencies
- Gated RNN
- RNN variants

# Loss Computation

Target outputs

$$L = \sum_t L^{(t)}$$



# Conditioned on Target Outputs

- Log-likelihood (cross-entropy)

$$-\log P\left(o^{(t)} = y^{(t)} \,\middle|\, x^{(1)},\ldots,x^{(t)},\ y^{(1)},\ldots,y^{(t-1)}\right)$$

Prediction at *t*

Target at *t*

# Conditioned on Target Outputs

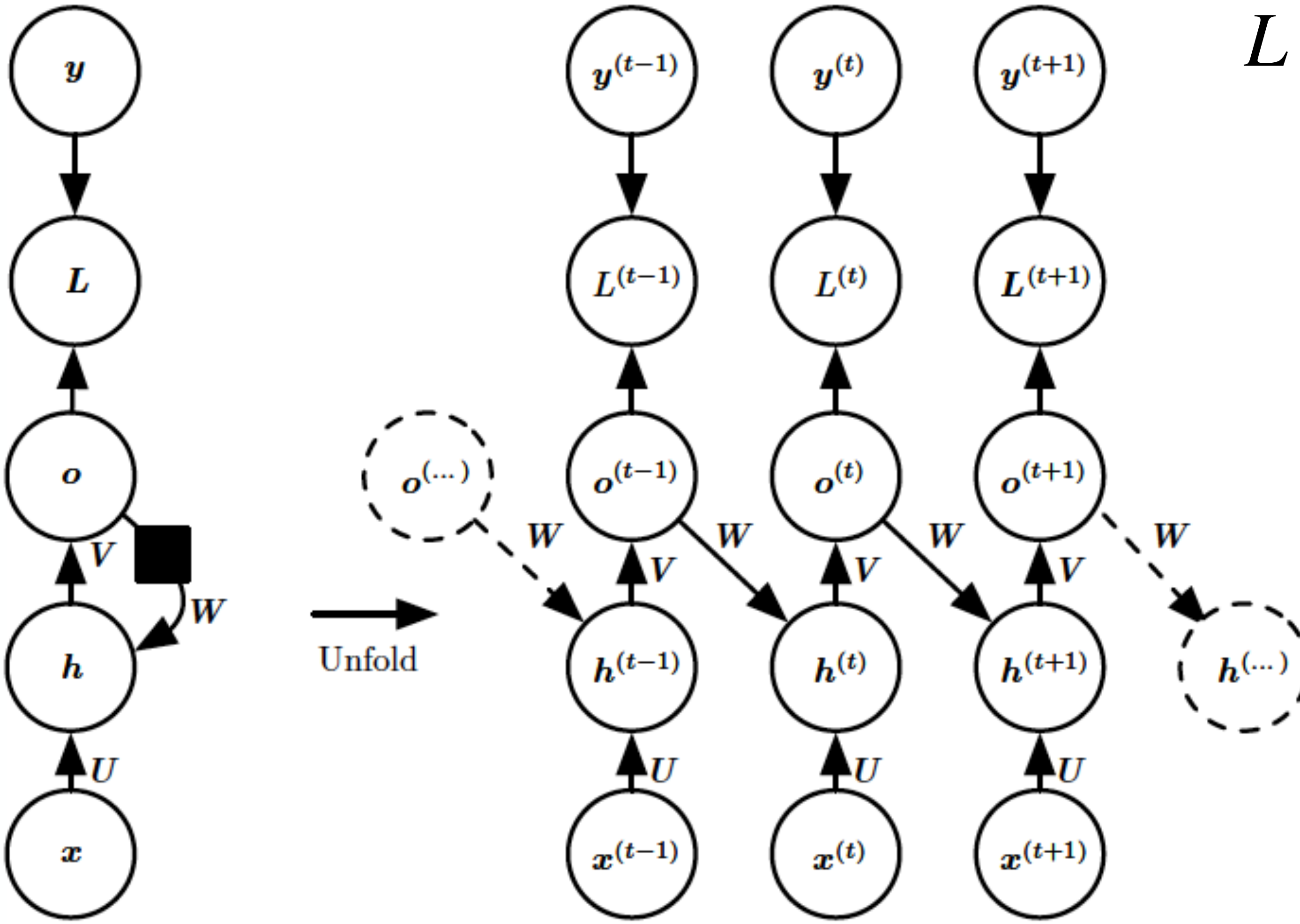
- Log-likelihood (cross-entropy)

$$-\log P\left(\underbrace{o^{(t)}}_{\text{Prediction at } t} = \underbrace{y^{(t)}}_{\text{Target at } t} \middle| x^{(1)},\ldots,x^{(t)},\ y^{(1)},\ldots,y^{(t-1)}\right)$$

- Conditioned on past inputs and outputs, output at time *t* is independent of future outputs

# Conditioned on Predicted Outputs

$$L = \sum_t L^{(t)}$$



# Conditioned on Predicted Outputs

- Log-likelihood

Past predictions
instead of true outputs

$$-\log P\left(o^{(t)} = y^{(t)} \,\middle|\, x^{(1)},\ldots,x^{(t)},\ \underleftrightarrow{o^{(1)},\ldots,o^{(t-1)}}\right)$$

Prediction at *t*

Target at *t*

# Conditioned on Predicted Outputs

- Log-likelihood

Past predictions
instead of true outputs

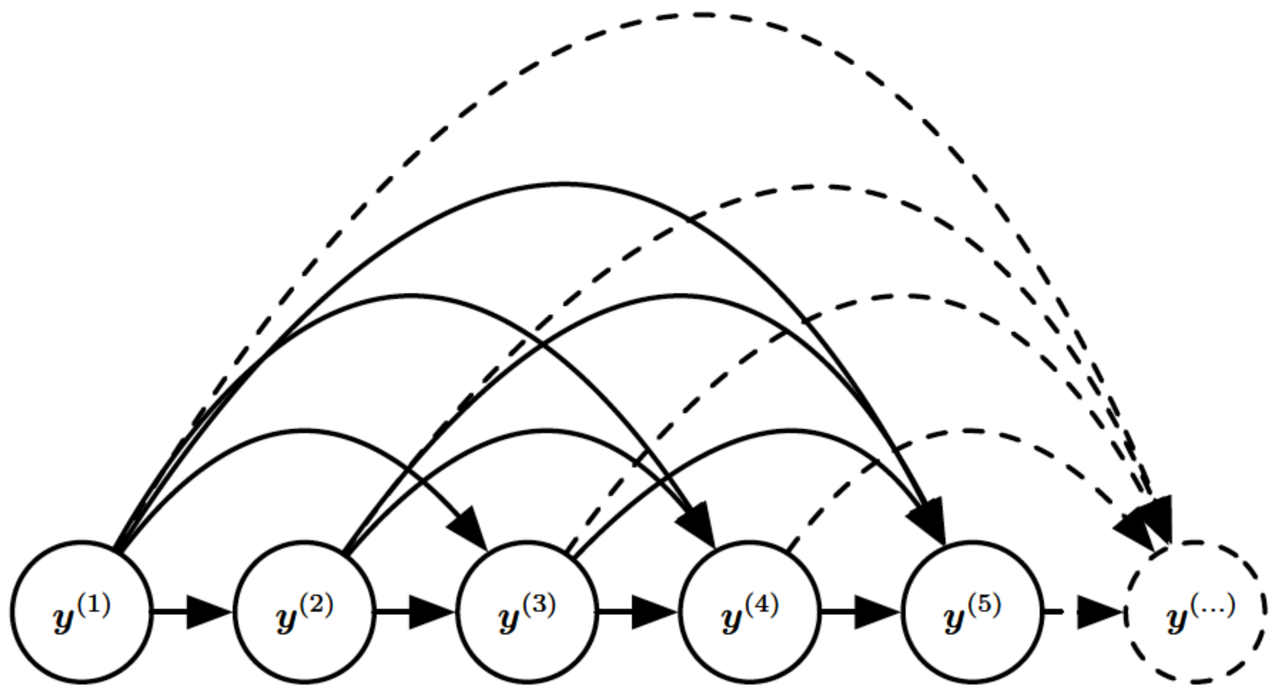
$$-\log P\left(o^{(t)} = y^{(t)} \,\middle|\, x^{(1)},\ldots,x^{(t)},\ \underleftrightarrow{o^{(1)},\ldots,o^{(t-1)}}\right)$$

Prediction at *t*          Target at *t*

- Conditioned on inputs, output at time *t* is independent of everything else

# Fully-connected graphical model

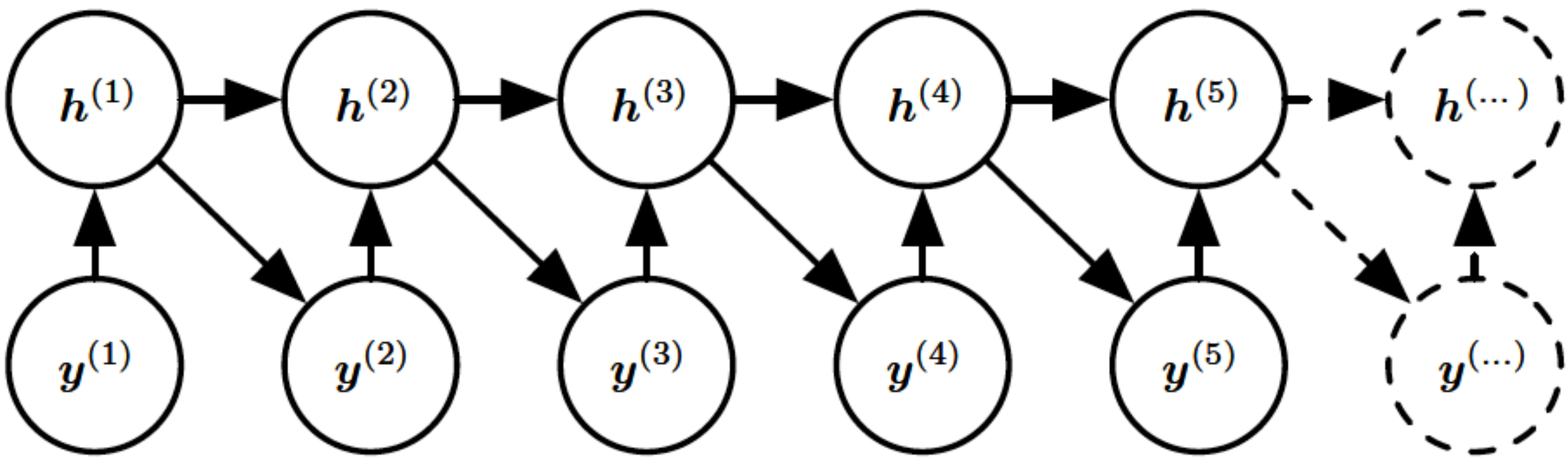
Simple example: *Predict day's stock prices based on previous prices*




*Inefficient* parametrization

# RNN graphical model

Simple example: *Predict day's stock prices based on previous prices*




*Efficient* parametrization,
but stationary distribution

# Outline

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- Gated RNN
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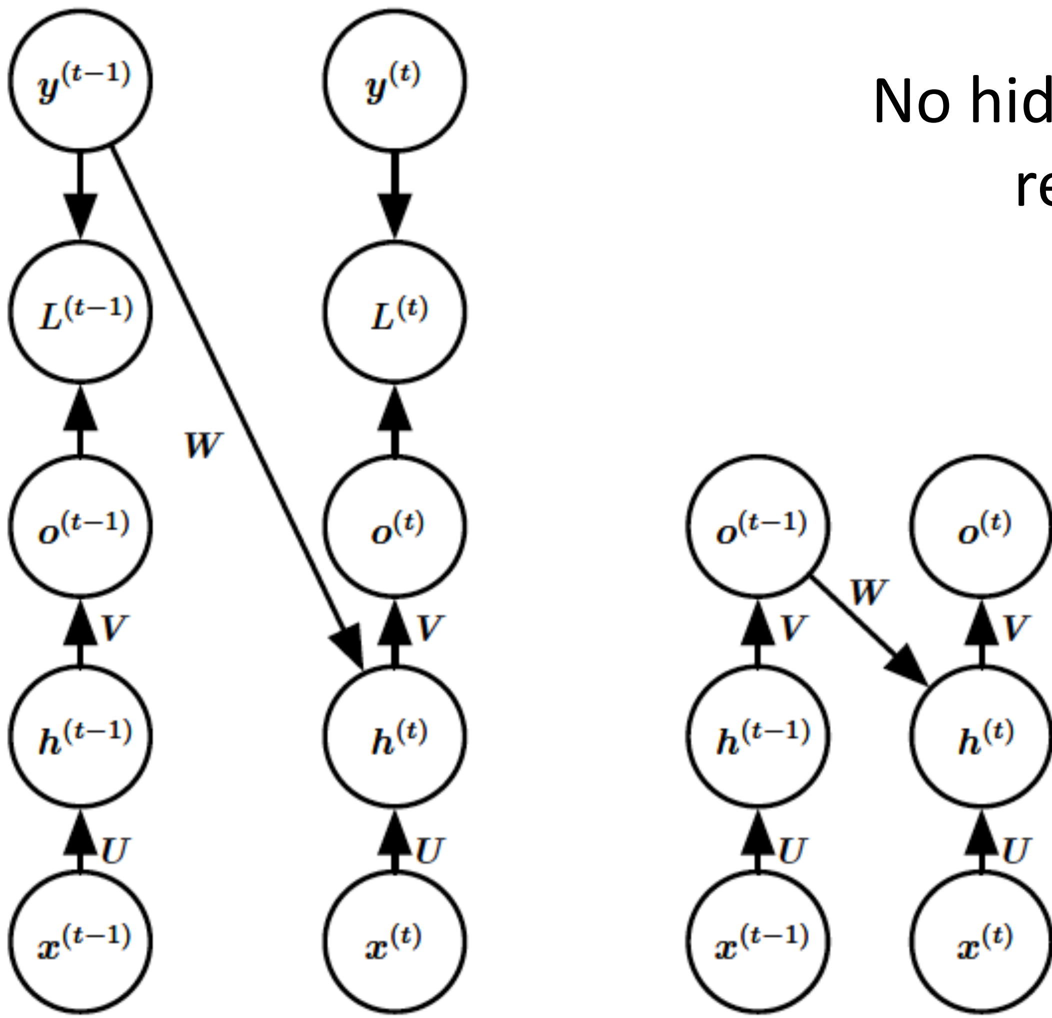
# Backprop Through Time

- For each input, unfold network for the sequence length $T$
- Back-propagation: apply forward and backward pass on unfolded network
- Memory cost: $O(T)$

# Case of Output Recurrence

No hidden-to-hidden recurrence




Train time

Test time

# Case of Output Recurrence

Loss at time $t$:

Teacher Forcing

$$L^{(t)} = -\log P\left(o^{(t)} = y^{(t)} \middle| x^{(1)},\ldots,x^{(t)},\ \underleftrightarrow{y^{(1)},\ldots,y^{(t-1)}}\right)$$

Use ground truth from previous time steps

Loss at different time steps are *decoupled*

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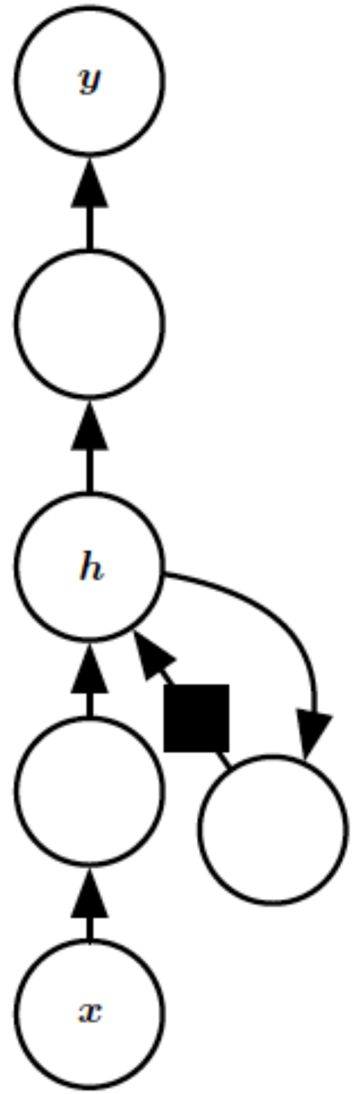
# Deep Recurrent Nets

Multiple layers between recurrent state and output




Multiple layers between input and recurrent state

Multiple layers between current and previous hidden states

# Long-term Dependencies

- Unfolded networks can be very deep
- Long-term interactions are given exponentially smaller weights than small-term interactions
- Gradients tend to either *vanish* or *explode*

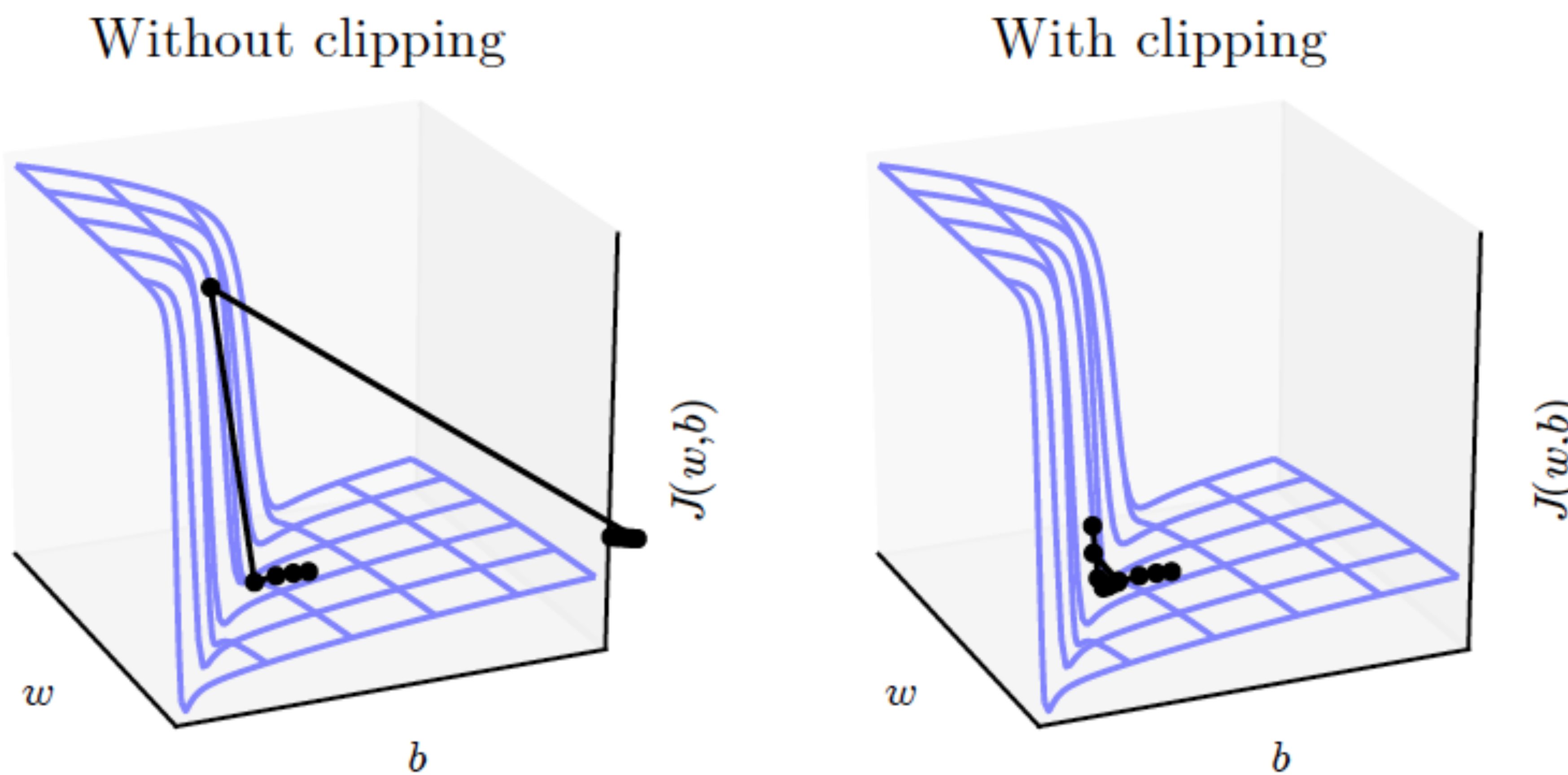
# Gradient Clipping

- Prevents exploding gradients
- Clip the norm of gradient before update:

$$\text{if } ||\boldsymbol{g}|| > v$$

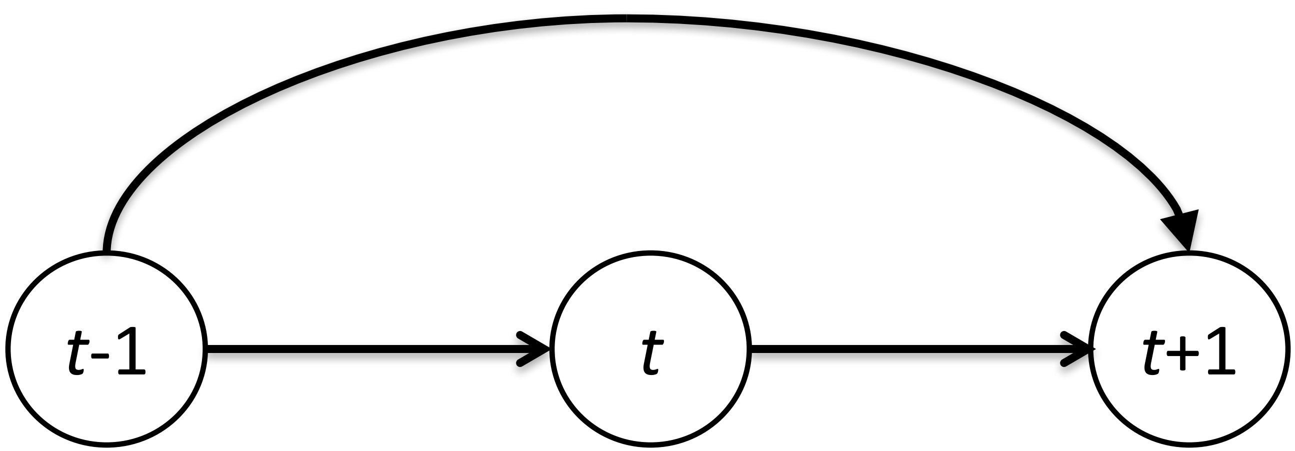
$$\boldsymbol{g} \leftarrow \frac{\boldsymbol{g}v}{||\boldsymbol{g}||}$$

# Gradient Clipping



# Skip Connections

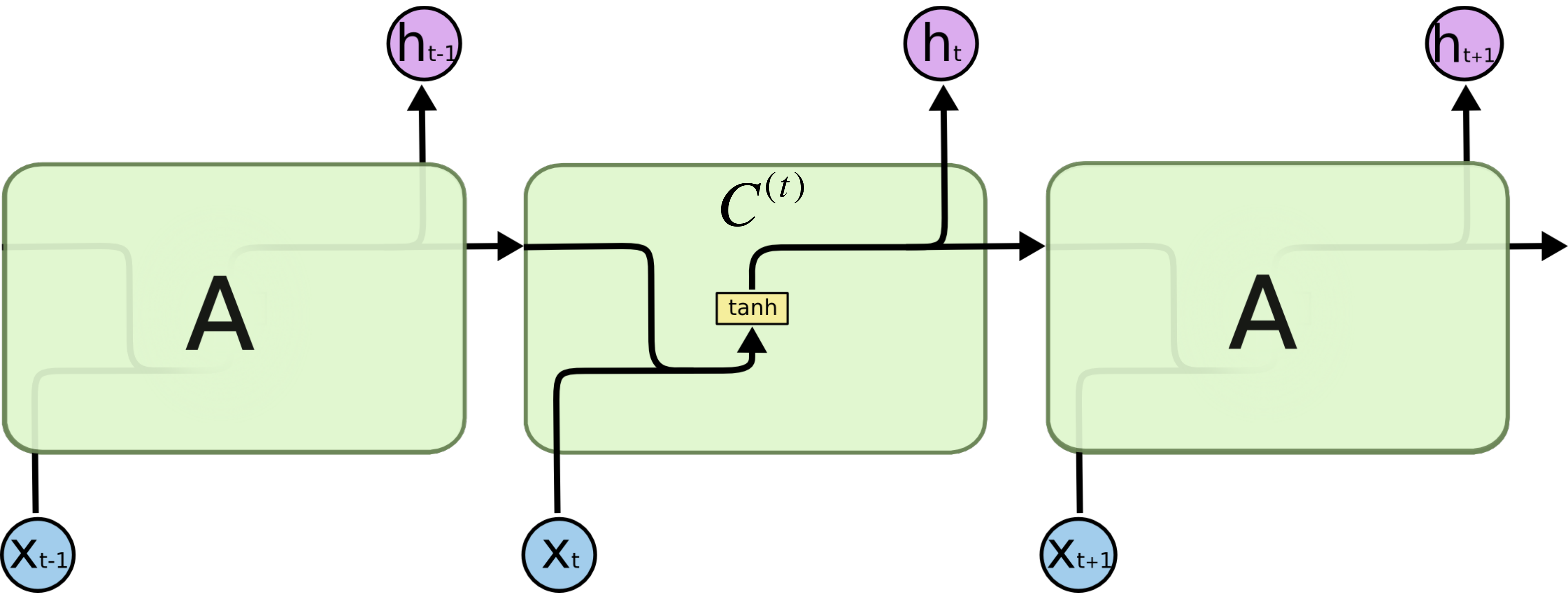
- Add additional connections between units *d* time steps apart
- Creating paths through time where gradients neither vanish or explode



# Leaky Units

- Linear self-connections
- Maintain cell state: running average of past hidden activations

# Standard RNN




$$C^{(t)} = \tanh(\mathbf{W}h^{(t-1)} + \mathbf{U}x^{(t-1)})$$

$$h^{(t)} = C^{(t)}$$

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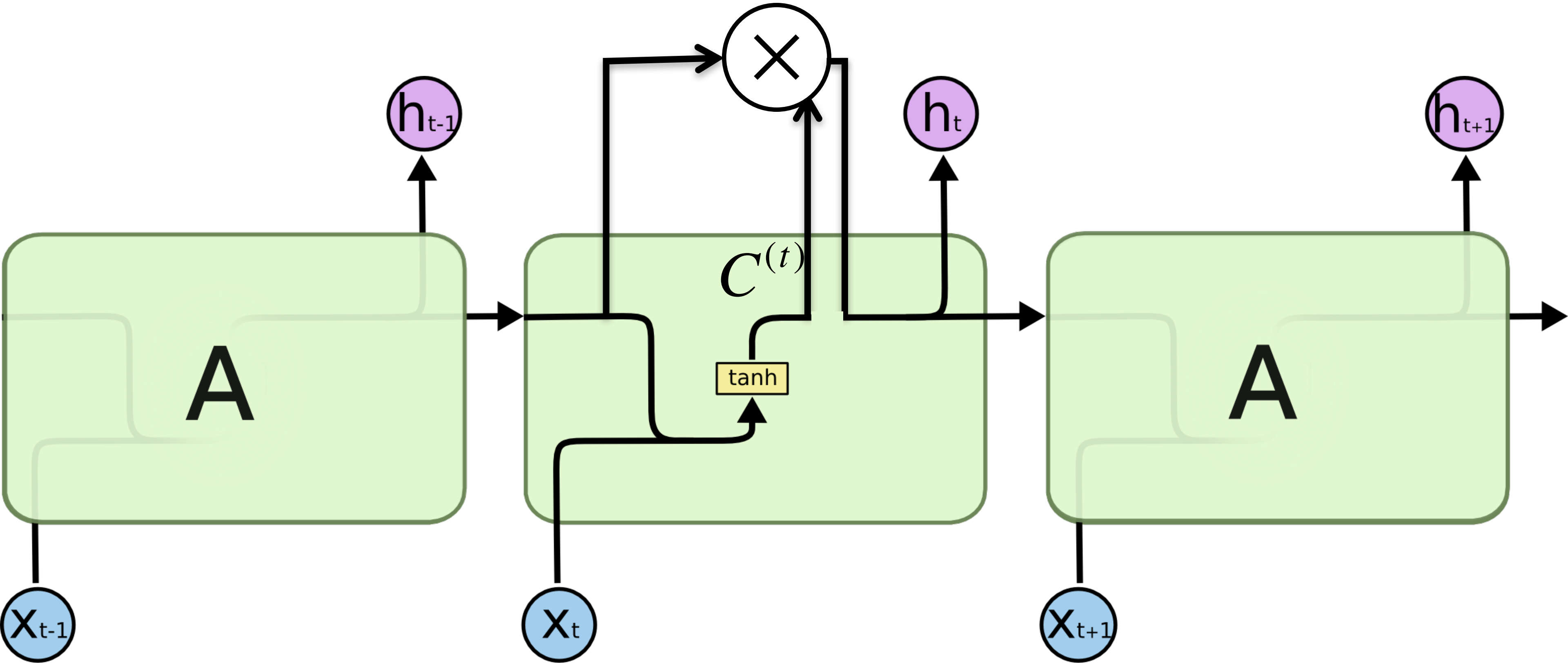
# Leaky Unit




$$C^{(t)} = \tanh(\mathbf{W}h^{(t-1)} + \mathbf{U}x^{(t-1)})$$

$$h^{(t)} = \alpha h^{(t-1)} + (1\text{-}\alpha)C^{(t)}$$

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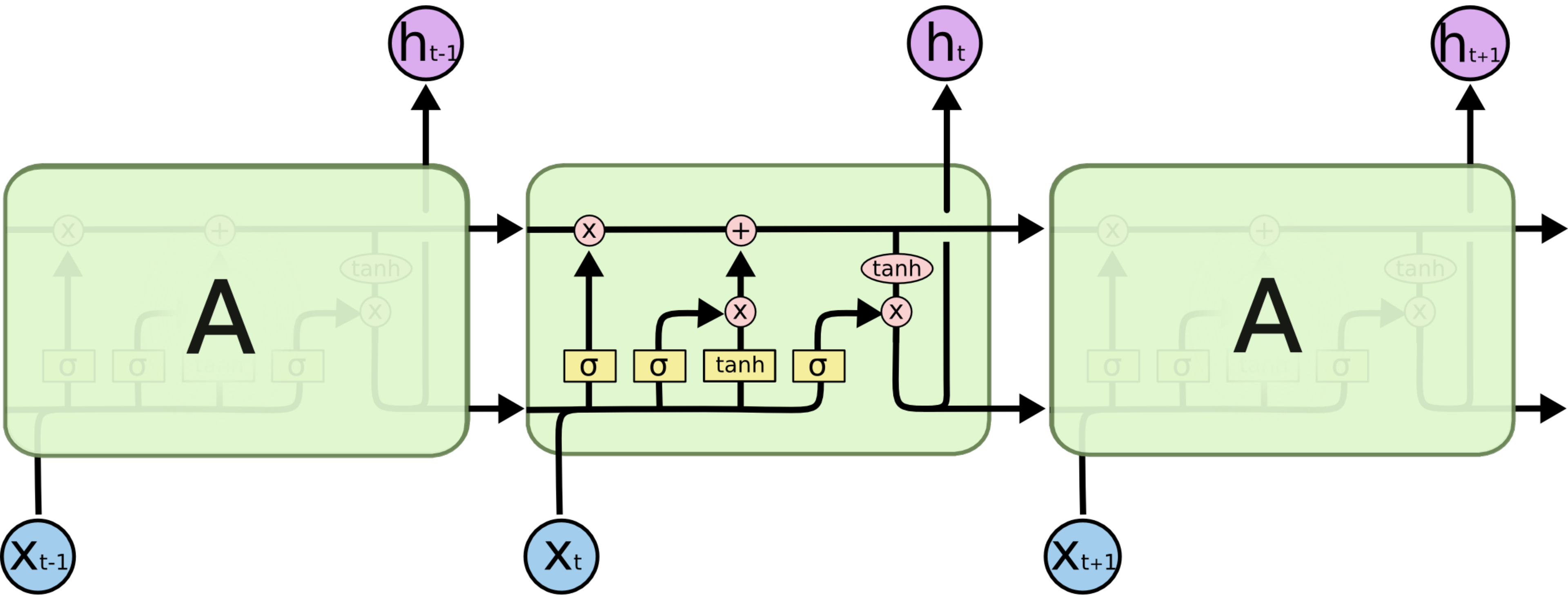
# Long Short-Term Memory

- Handles long-term dependencies
- Leaky units where weight on self-loop $\alpha$ is context-dependent
- Allow network to decide whether to accumulate or forget past info

# Long Short-Term Memory

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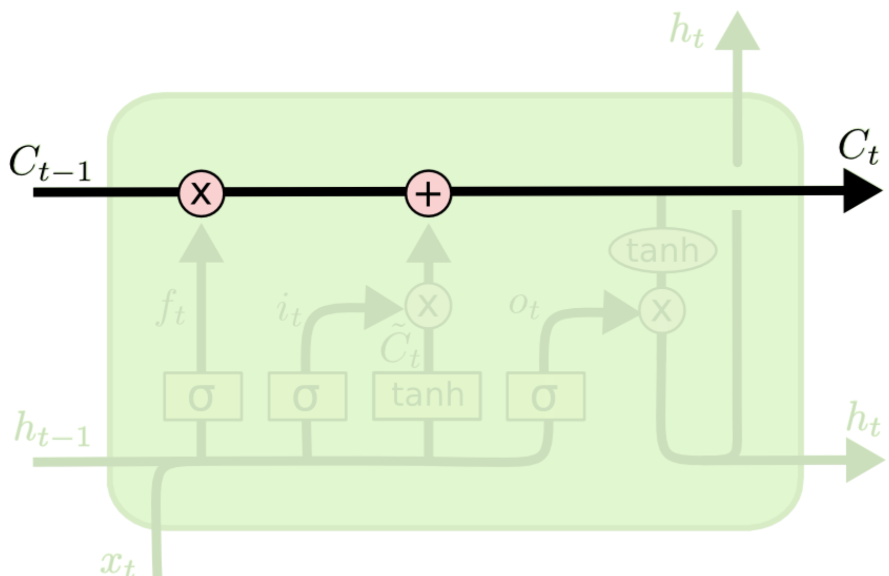
# Cell State

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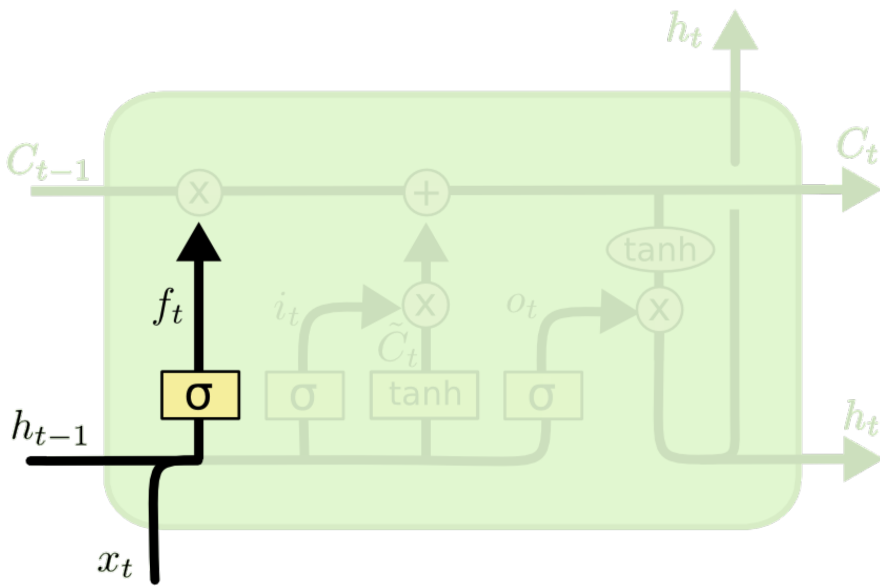
# Forget Gate




$$f^{(t)} = \sigma(W^f h^{(t-1)} + U^f x^{(t)})$$

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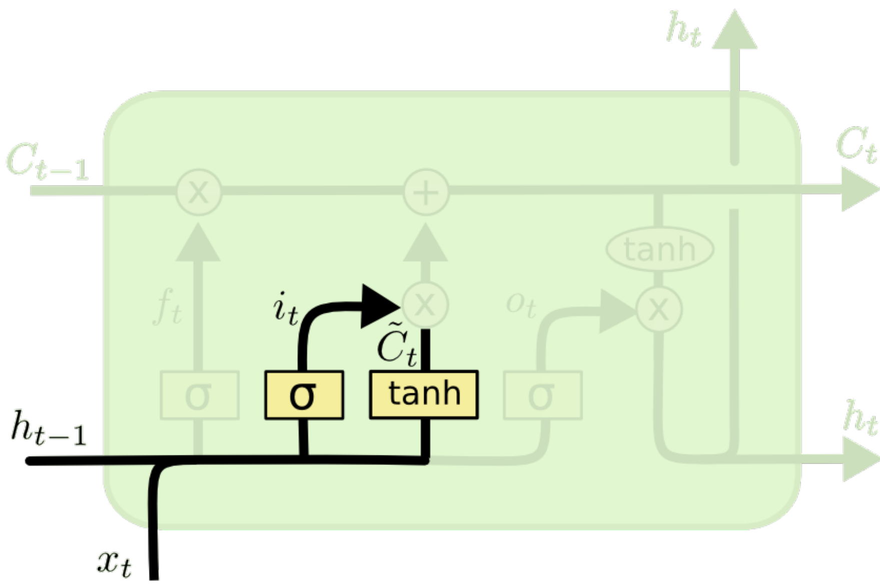
# Input Gate




$$i^{(t)} = \sigma(W^i h^{(t-1)} + U^i x^{(t)})$$

$$\tilde{C}^{(t)} = \tanh(W h^{(t-1)} + U x^{(t)})$$

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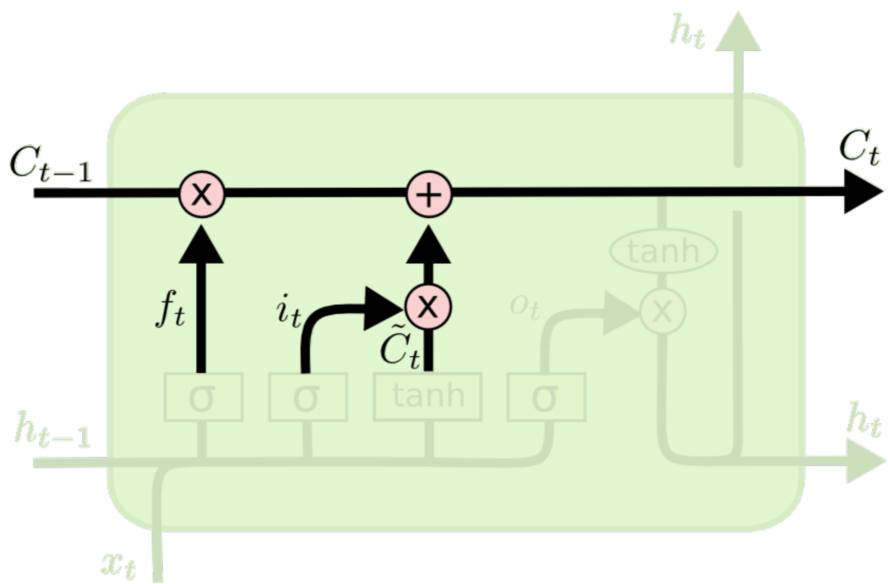
# Cell State Update




$$C^{(t)} = f^{(t)} C^{(t-1)} + i^{(t)} \tilde{C}^{(t)}$$

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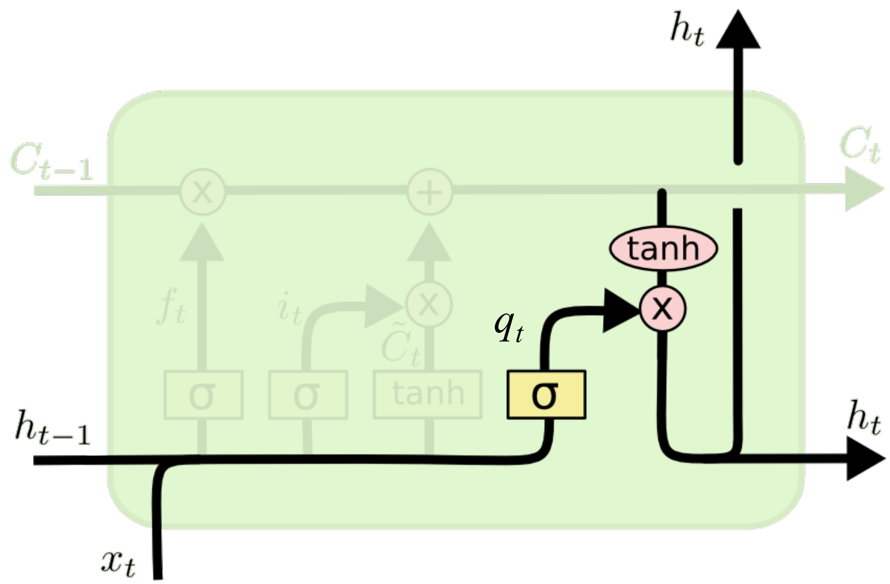
# Output Gate




$$q^{(t)} = \sigma(W^o h^{(t-1)} + U^o x^{(t)})$$

$$h^{(t)} = \tanh(C^{(t)})$$

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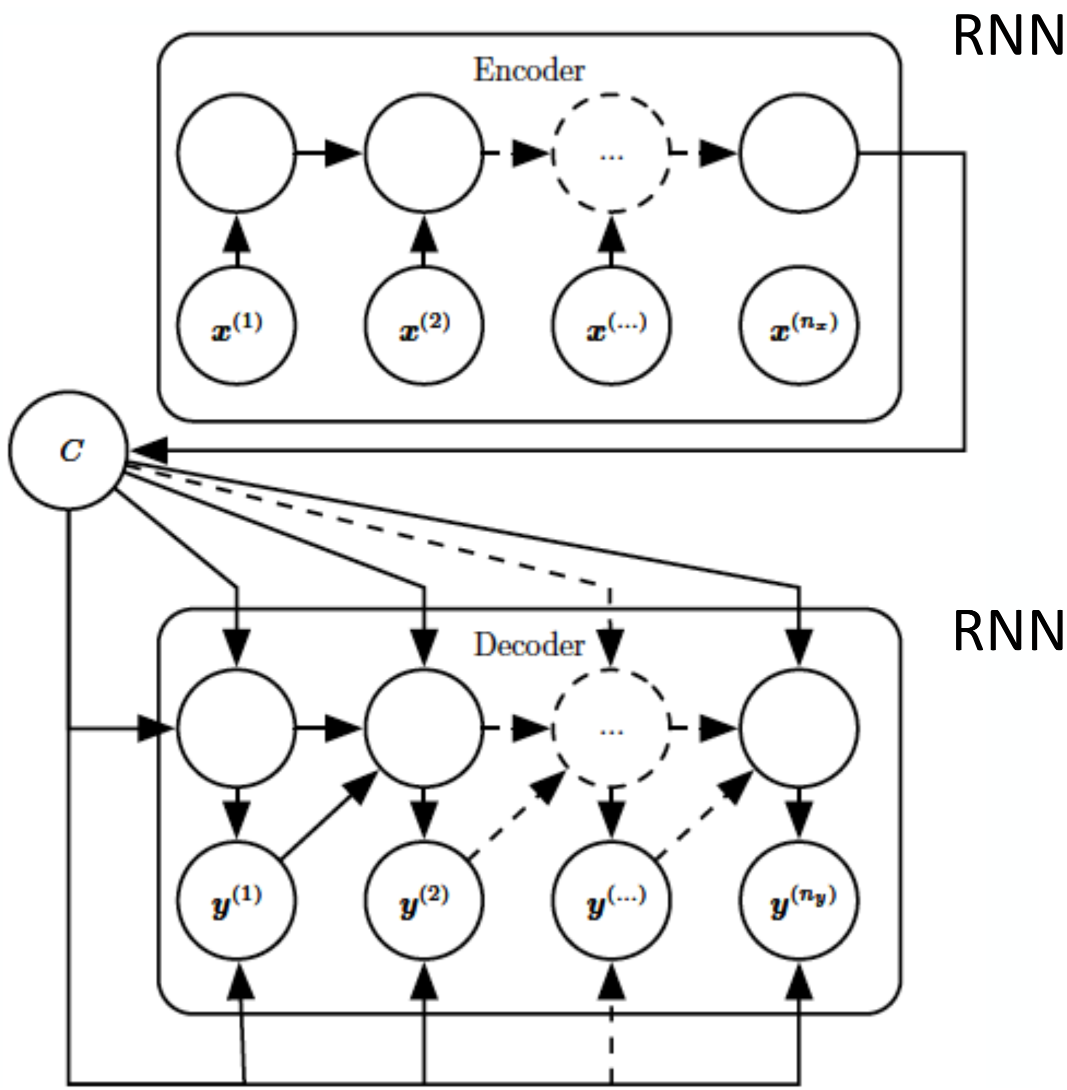
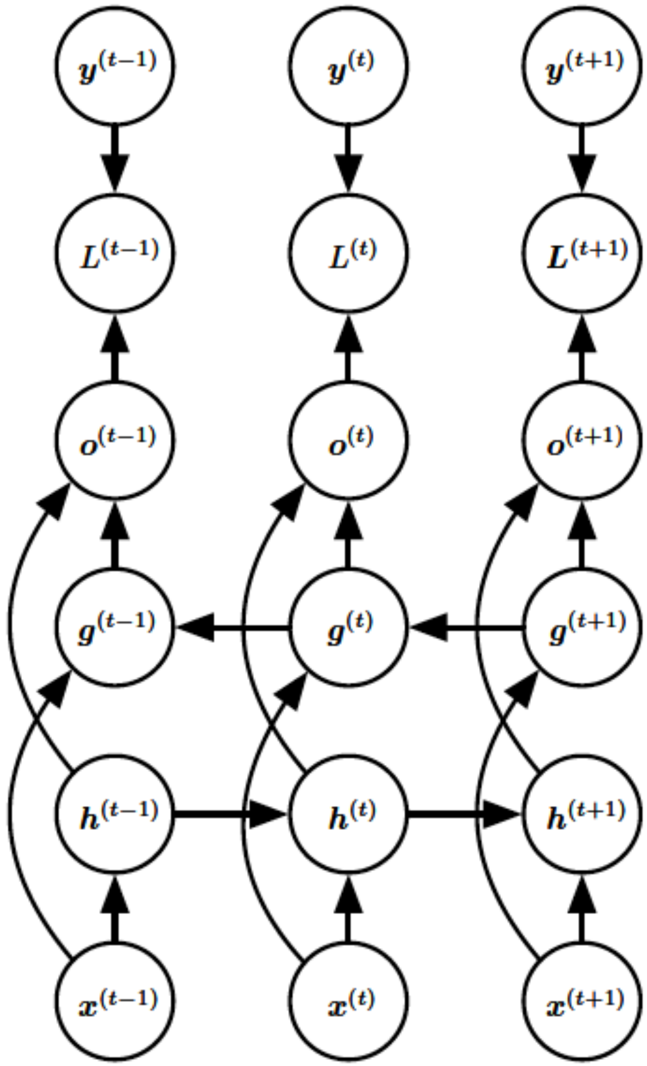
# Encoder-decoder Networks

RNN output sequence can be of different length than input sequence



# Bidirectional Network

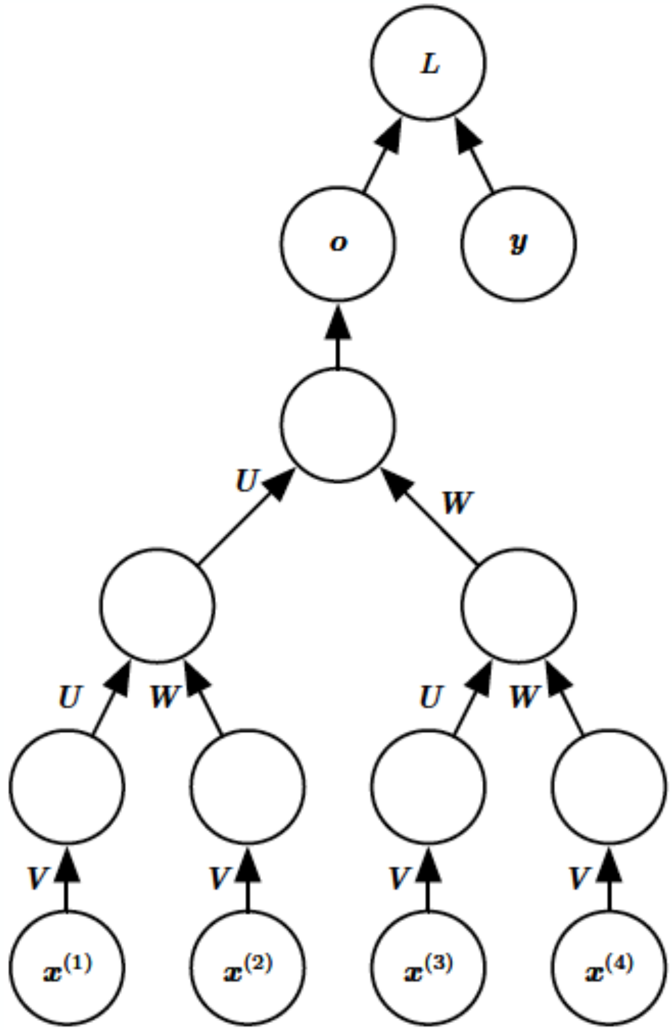



Output prediction may depend on *whole input sequence*

E.g. speech recognition: current sound may depend on future phonemes

## Backprop?

# Recursive Network




Tree structure vs. chain
E.g. parse tree in NLP

Reduce network depth by using taller trees