Stan Lab

CS 109b Staff
2/21/18 – 2/22/18

Motivation

- How do we normalize posteriors?

Normalization Constant Example:

Given a dataset $D = [x_1, x_2, ...x_n]$

Bayes Rule tells us that:

$P(\theta \mid D) = \frac{P(D|\theta)P(\theta)}{P(D)}$, where

$P(\theta)$ is our prior distribution,
$P(D \mid \theta)$ is our likelihood function,
$P(\theta \mid D)$ is our posterior distribution

Problem: How do we calculate $P(D)$?

$P(D) = \int P(D, \theta) d\theta$,
which can be very high-dimensional and difficult to compute.

Motivation

- How do we normalize posteriors?
  - Conjugate priors
  - Metropolis-Hastings
  - Other forms of MCMC
- Strong developments in both sampling algorithms and computational power have made Bayesian models more feasible

Background

- Stan was developed by Andrew Gelman (PhD from Harvard in 1990) and his lab at Columbia University
  - BDA
- Many predecessors to Stan such as Bugs, Jags, etc.
- Landmark paper: *The No-U-Turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo*
  - Inspired by ideas from physics

Bayesian Workflow

"How to structure the process of your analysis to maximise [sic] the odds that you build useful models."

-Jim Savage

Sean Talts
Core Stan Developer

Bayesian Workflow

1. **Scope out your problem**
   What inputs and outputs can help you learn? What relationships can you see by eye?
2. **Specify likelihood & priors**
   Use knowledge of the problem to construct a generative model and shape the scope of the parameters
3. **Check the model with fake data**
   Generate data, fit model, and evaluate fit as a sanity check
4. **Fit the model to real data**
   To recover parameters
5. **Check diagnostics**
   Algorithms should come with diagnostics that let you know when they're not working
6. **Graph fit estimates**
   Understand your inferences
7. **Check predictive posterior**
   Perform PPCs to understand predictions
8. **Compare models**
   Iterate on model design, choose a model

Modeling Heights and Weights

$$height \sim N(\alpha + \beta * weight, \sigma^2)$$

Modeling Heights and Weights

- In Stan, code is structured in blocks
  - Functions
  - Data
  - Transformed Data (optional)
  - Parameters
  - Transformed Parameters (optional)
  - Model
  - Generated Quantities (optional)

Modeling Heights and Weights

- In Stan, code is structured in blocks
  - Functions
  - **Data**

```
data {
  int num_people;
  vector<lower=0>[num_people] weights;
  vector<lower=0> heights[num_people];
}
```

Notice how variables are statically-typed, rather than the dynamically-typed format you've used in R and in Python. You'll also have to specify whenever you want to end a line with a semicolon.

Stan also allows you to bound both data and parameters.

Modeling Heights and Weights

- In Stan, code is structured in blocks
  - Functions
  - Data
  - **Parameters**

```
parameters {
  real beta;
  real alpha;
  real<lower=0> sigma;
}
```

Modeling Heights and Weights

- In Stan, code is structured in blocks
  - Functions
  - Data
  - Parameters
  - **Model**

```
model {
  heights ~ normal(beta * weights + alpha, sigma);
}
```

Modeling Heights and Weights

- In Stan, code is structured in blocks
  - Functions
  - Data
  - Parameters
  - **Model**

```
model {
  heights ~ normal(beta * weights + alpha, sigma);
  beta ~ normal(0, 10); // cm/kg
  alpha ~ normal(50, 50); // avg cm for 0 kg
  sigma ~ normal(0, 5); // variation from average
}
```

Modeling Heights and Weights

- After we fit our model, we can perform a sanity check
  - 1) Draw parameter values from the posterior
  - 2) Generate data based on those parameter values
  - 3) Fit model to generated data
  - 4) Check if fit is reasonable

```
generated quantities {
  real<lower=0> heights[N];
  real beta = normal_rng(0, 10);
  real alpha = normal_rng(50, 50);
  real sigma = fabs(normal_rng(0, 5));
  for (n in 1:N)
    heights[n] = normal_rng(beta * weights[n] + alpha, sigma);
}
```

Your Turn!

- Open a blank R script and source the file "count_data.R", which is provided in the Lab materials
- Make sure you have the "rstan", "ggplot2", and "bayesplot" packages installed
- Try your best to fit the model described by the instructor